K-SPACE AND IMAGE RECONSTRUCTION

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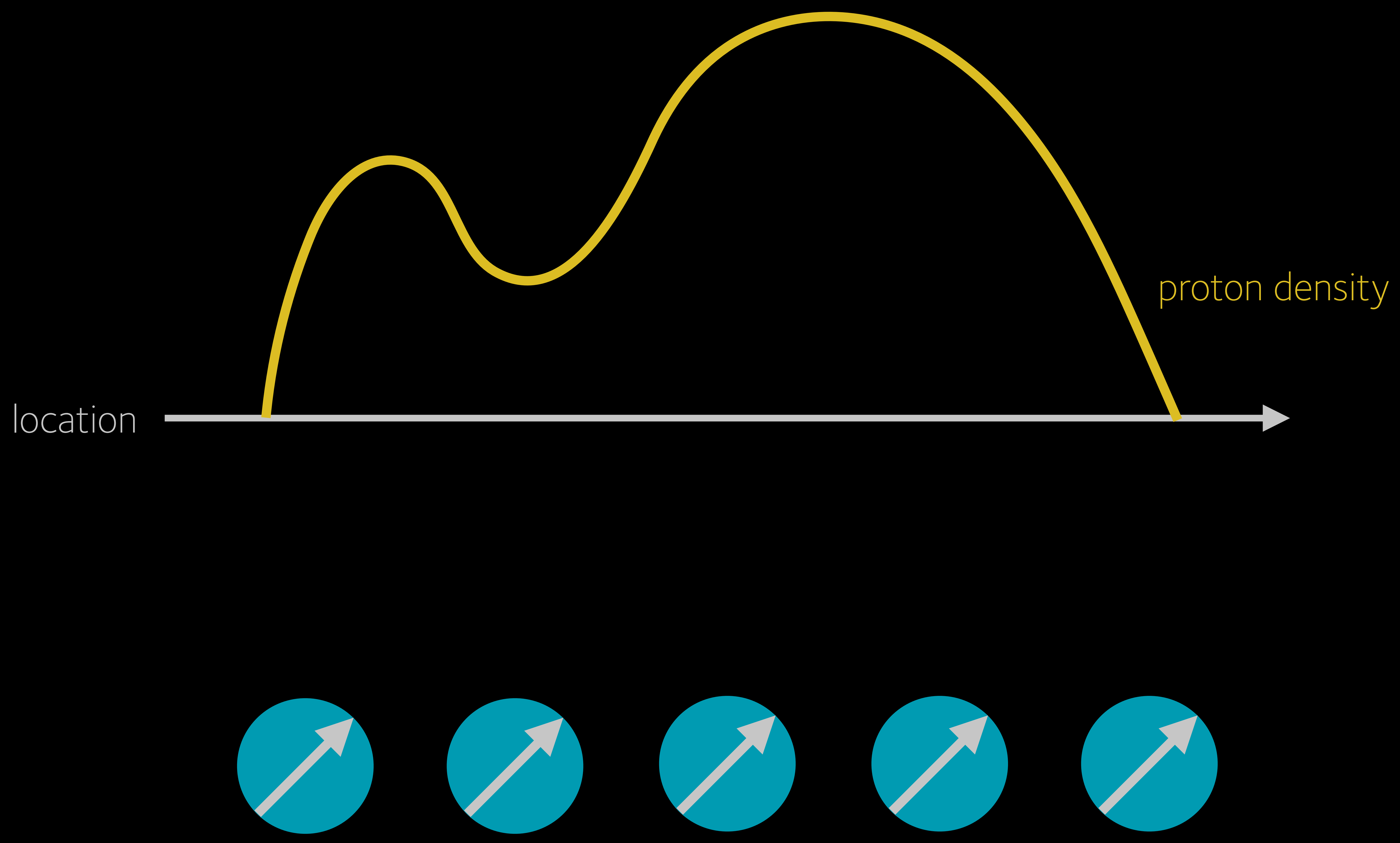
OUTLINE

- overview of MR signal sampling and k-space
- fundamentals of image reconstruction
- non-cartesian acquisitions
- accelerated MRI
  - partial Fourier
  - parallel imaging
    - SENSE
    - GRAPPA

SPATIAL ENCODING



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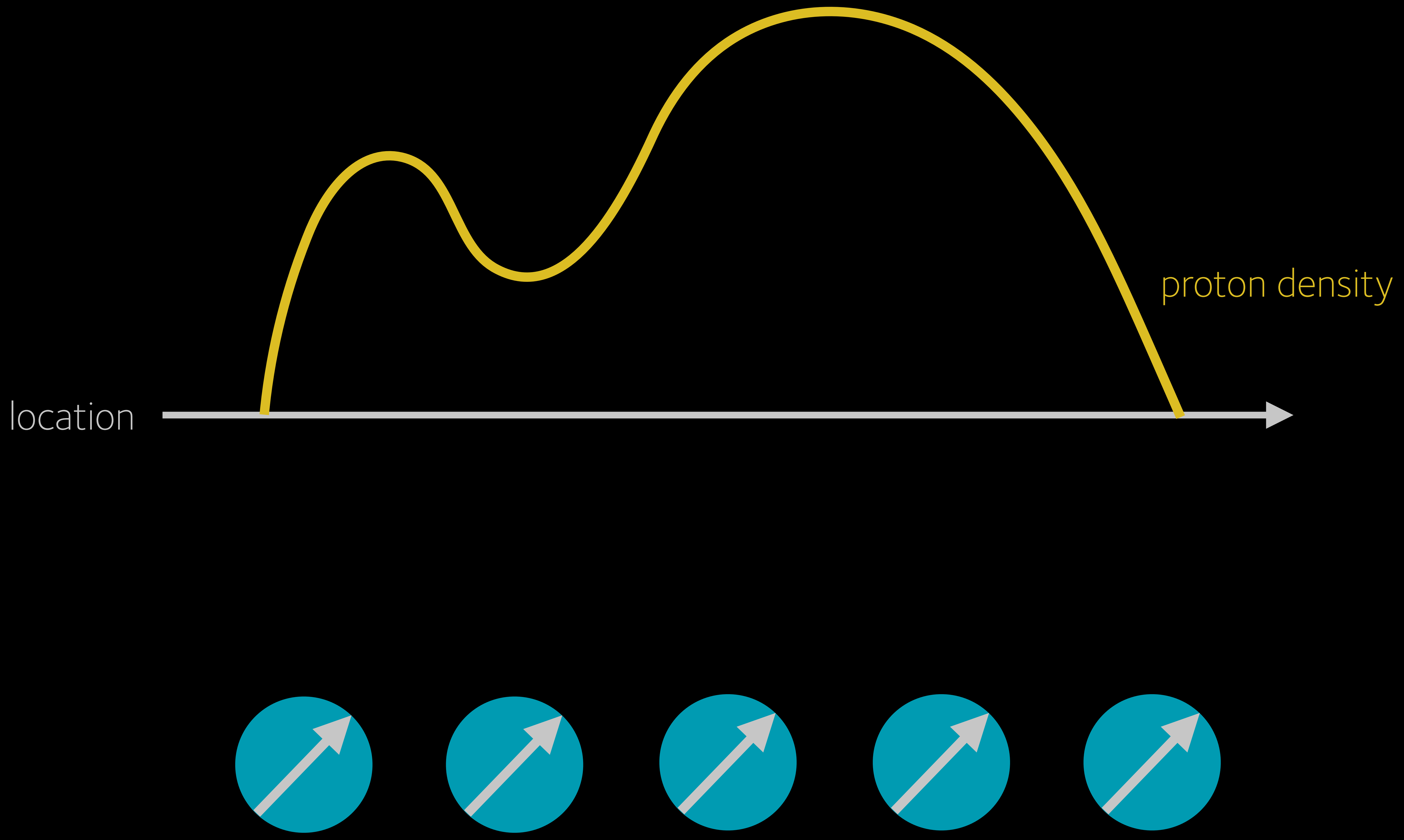

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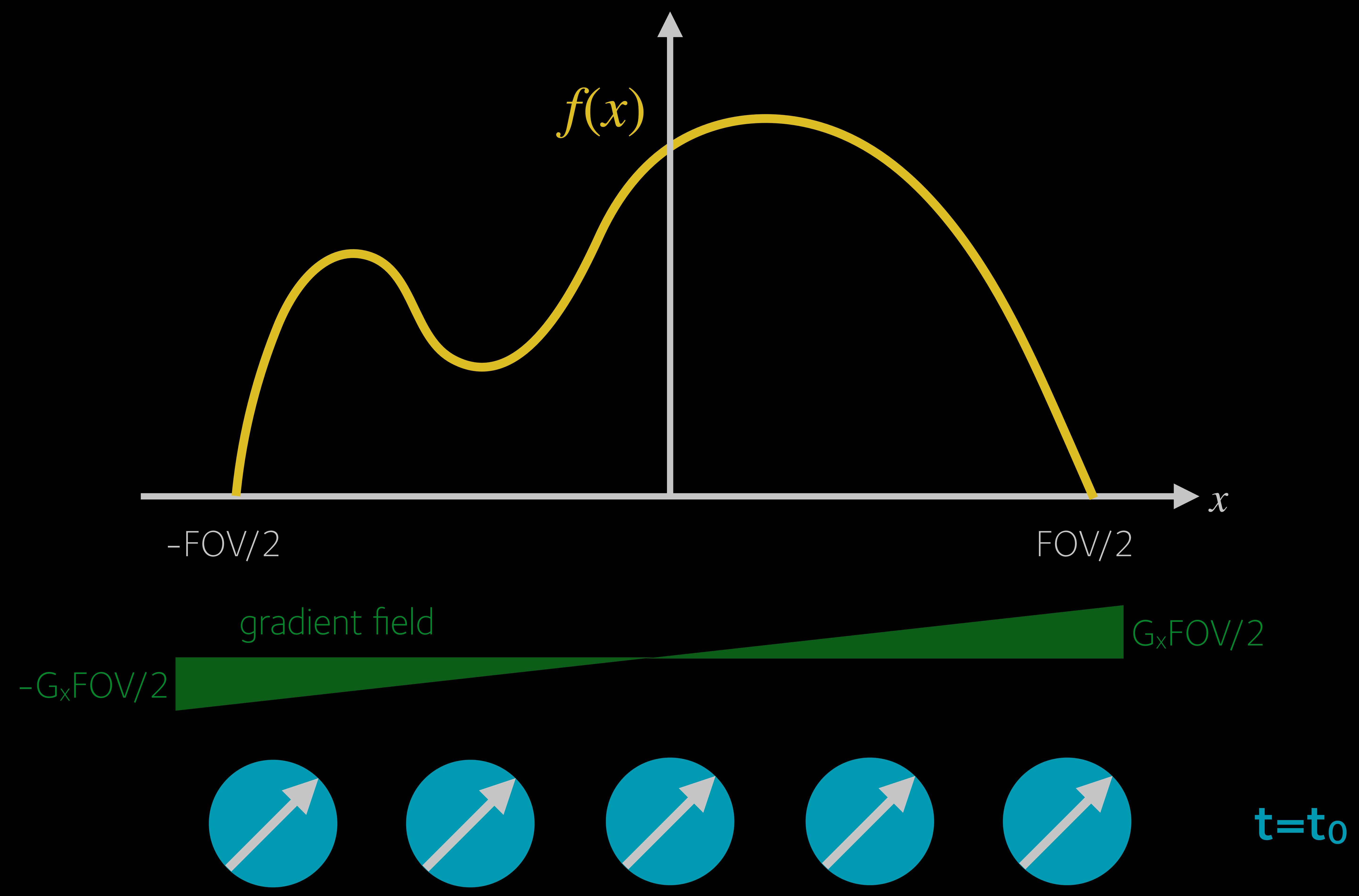

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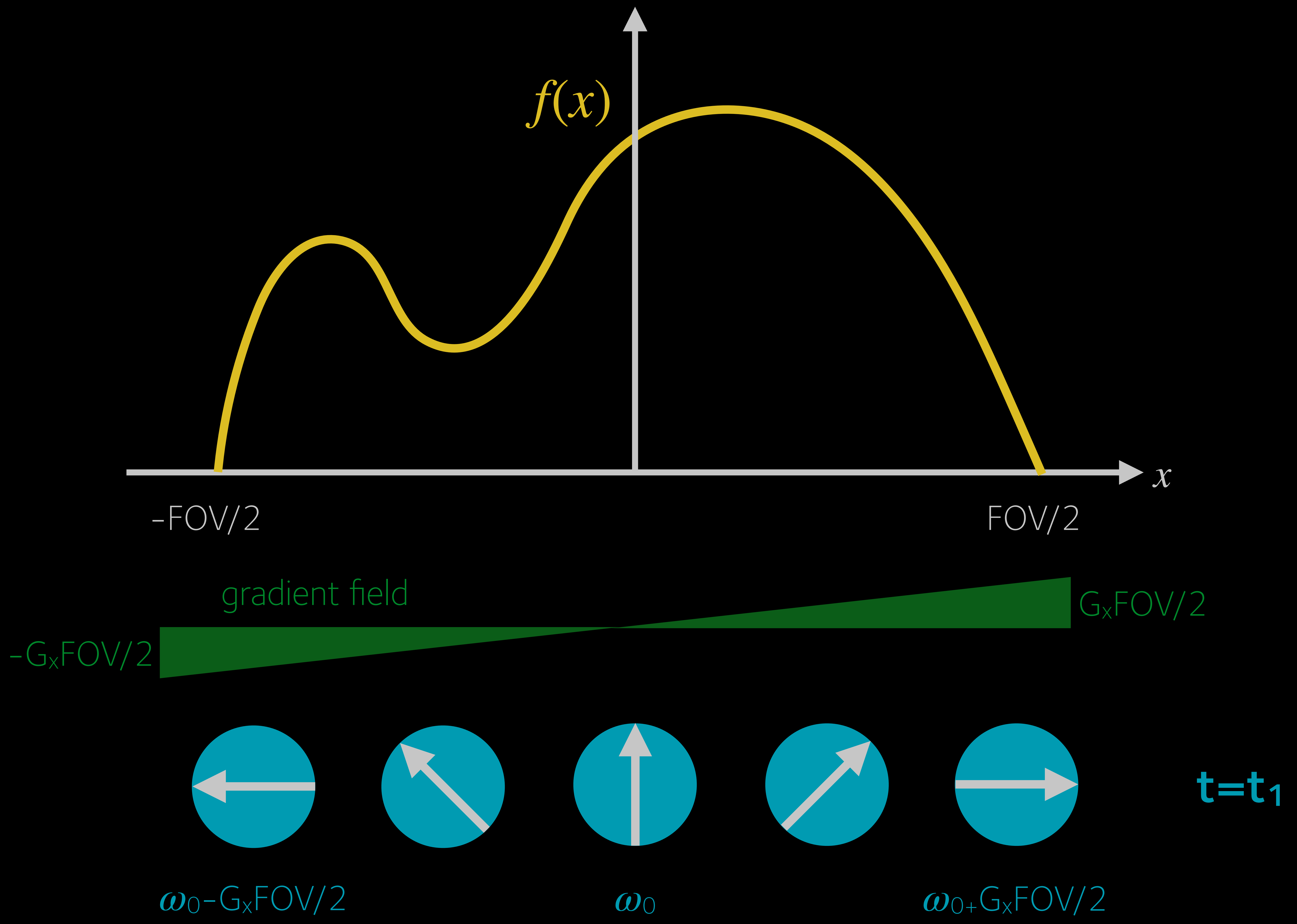
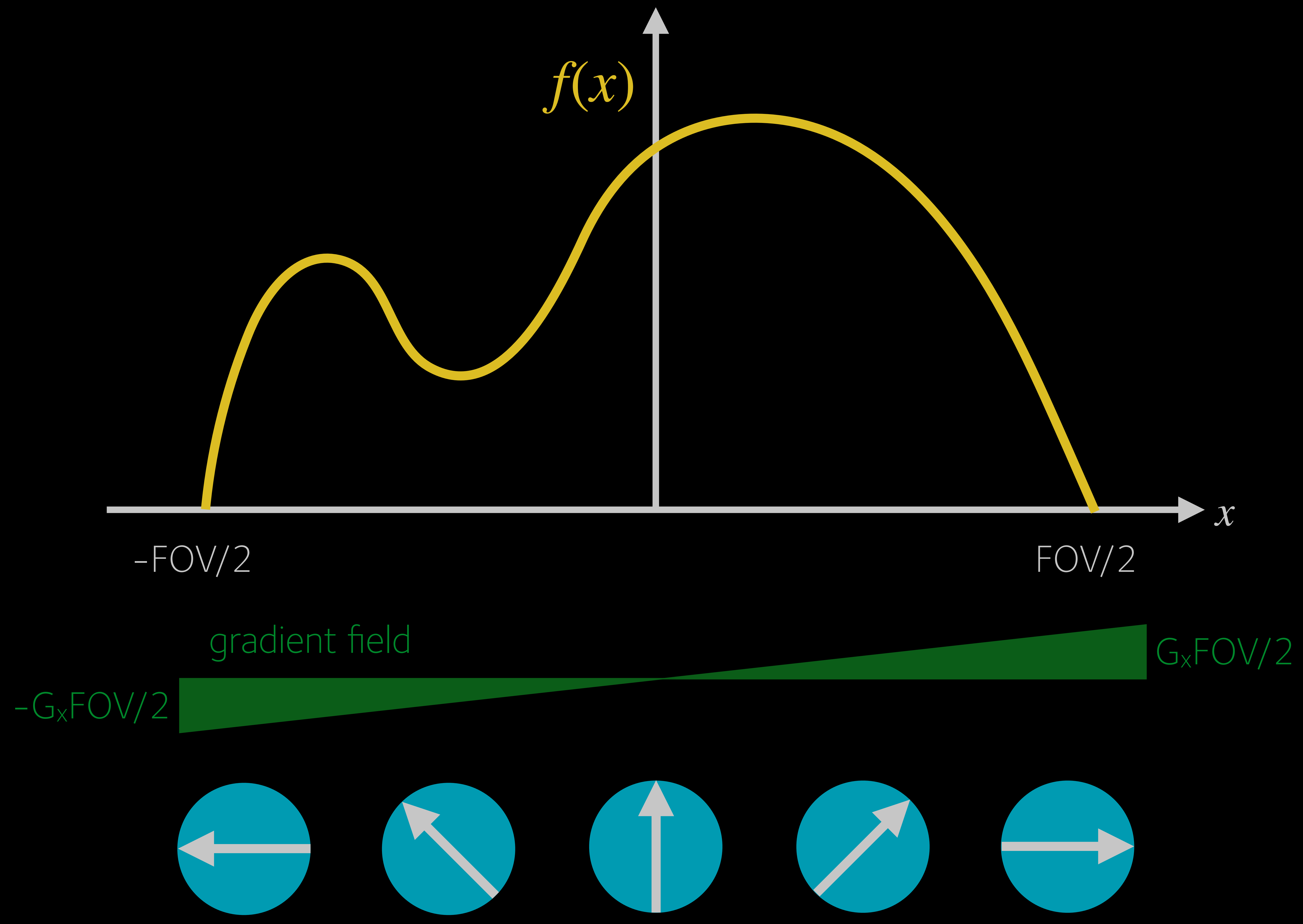
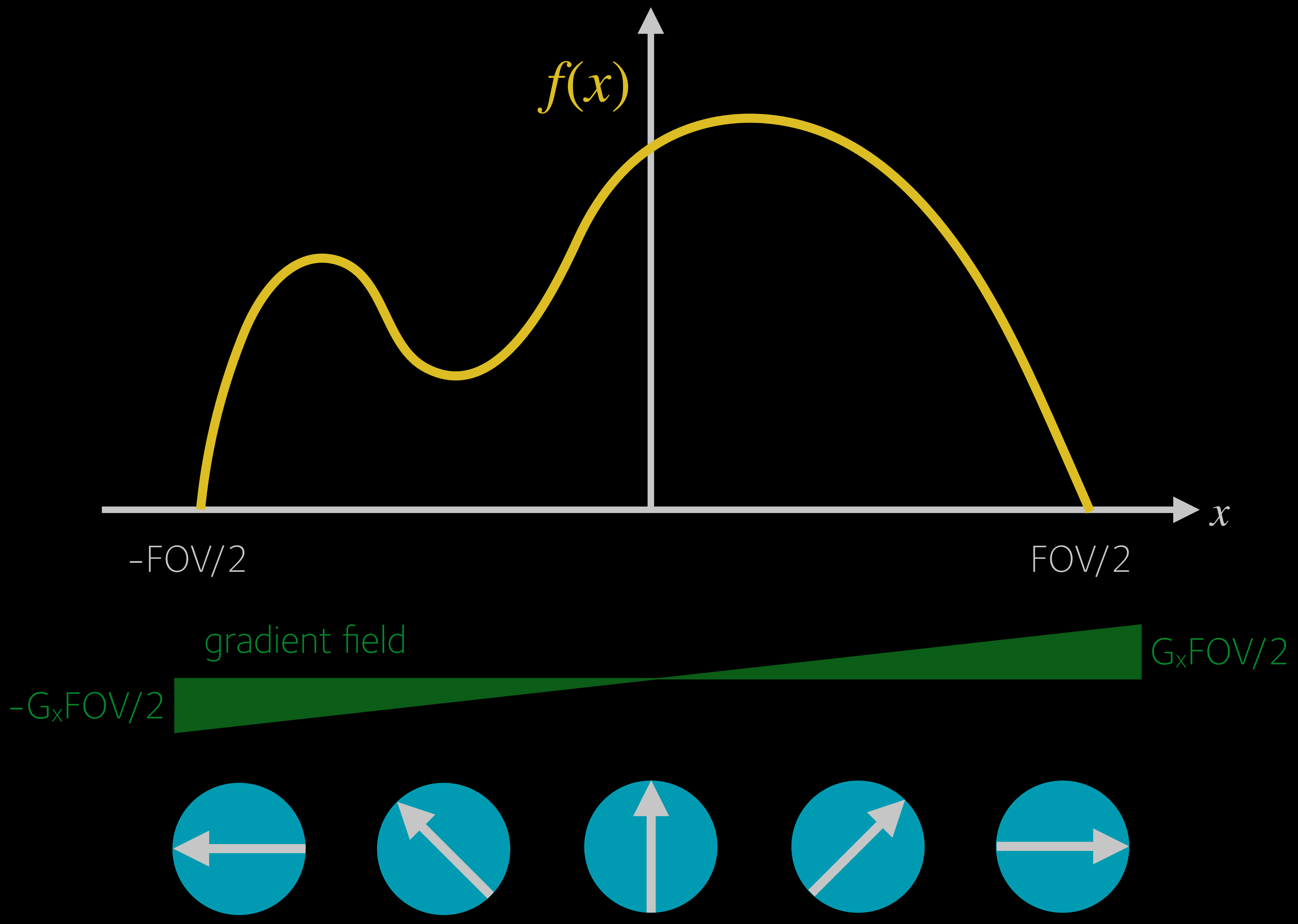


MR SIGNAL



$$s(t) = \int f(x) e^{-i\Phi(x,t)} dx$$

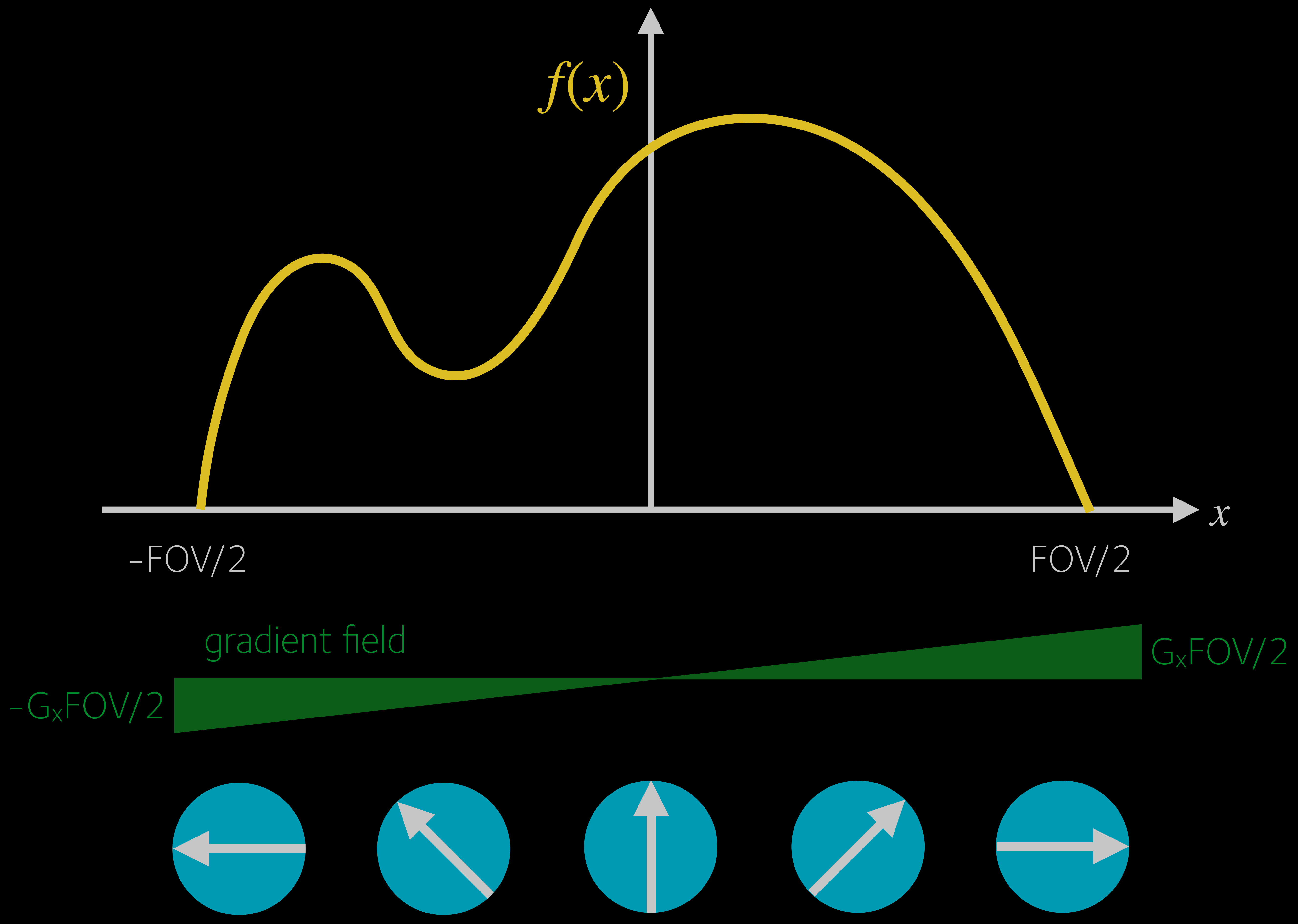
MR SIGNAL



$$s(t) = \int f(x) e^{-i\Phi(x,t)} dx$$

$$\Phi(x,t) = \gamma G_x t x = 2\pi \underbrace{\left(\frac{\gamma}{2\pi} G_x t \right)}_{k_x(t)} x$$

MR SIGNAL



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$$s(t) = \int f(x) e^{-i2\pi k_x(t) x} dx$$

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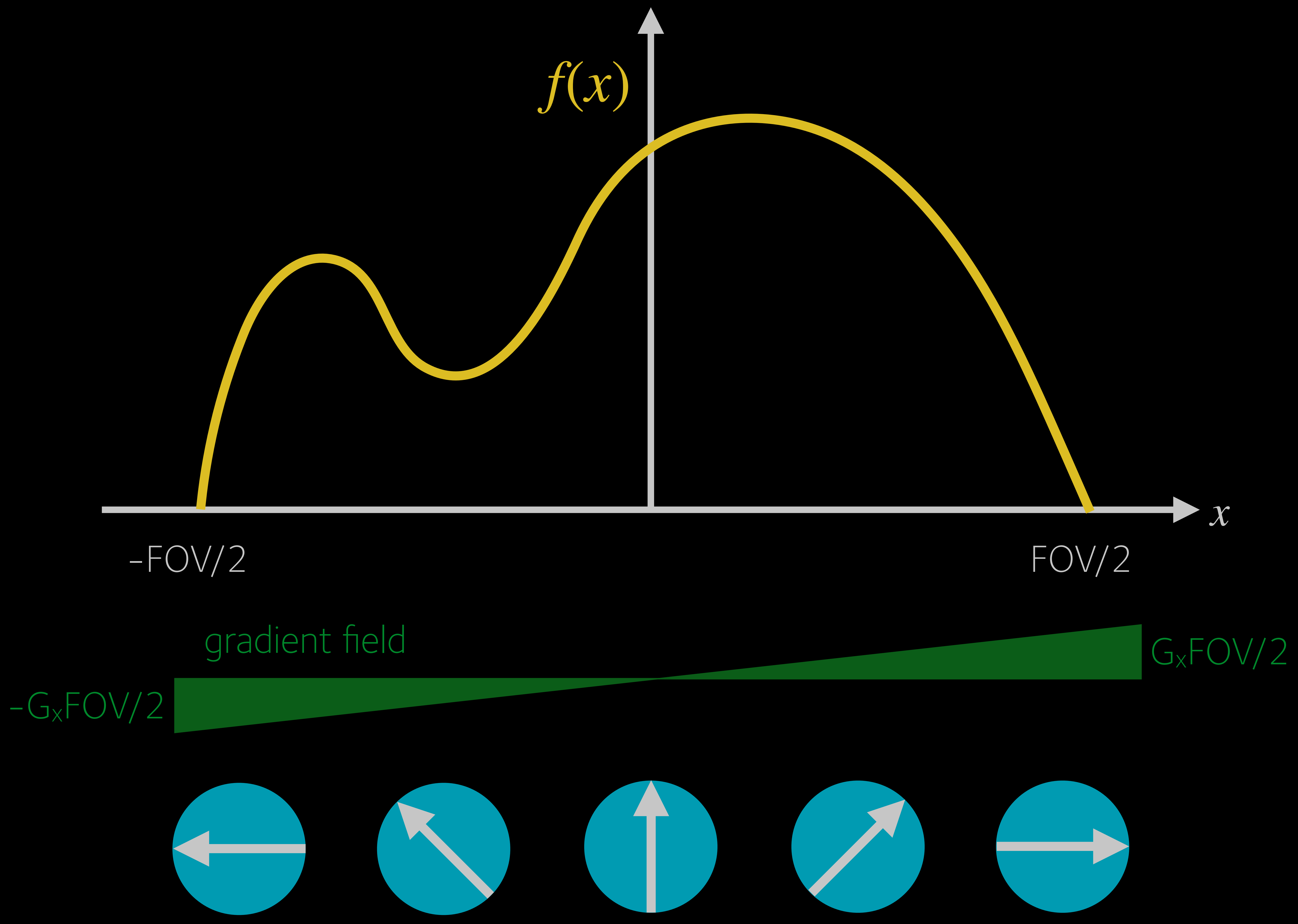

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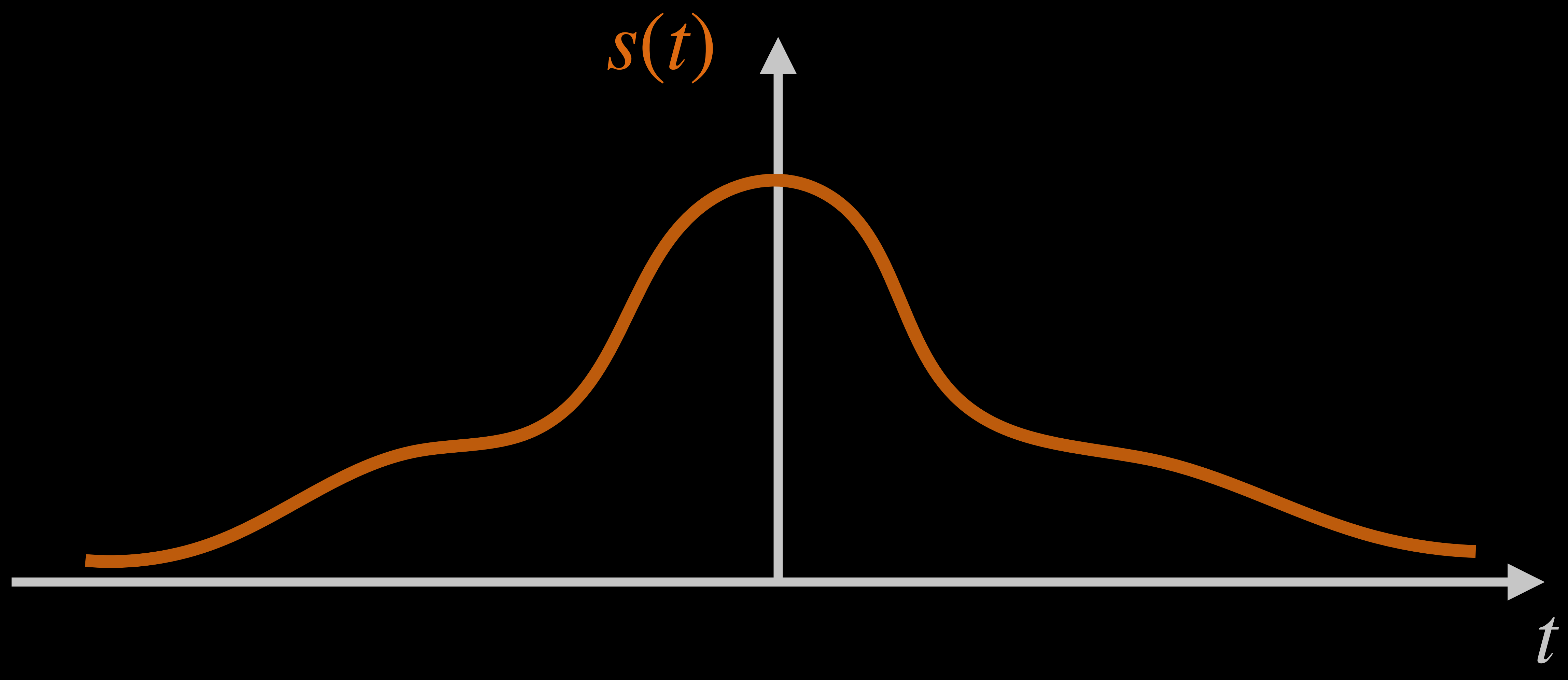
$$\Phi(x,t) = \gamma G_x t x = 2\pi \underbrace{\left(\frac{\gamma}{2\pi} G_x t\right)}_{k_x(t)} x$$

$$s(t) = \int f(x) e^{-i2\pi k_x(t) x} dx = \mathscr{F}\{f(x)\}$$

SIGNAL SAMPLING



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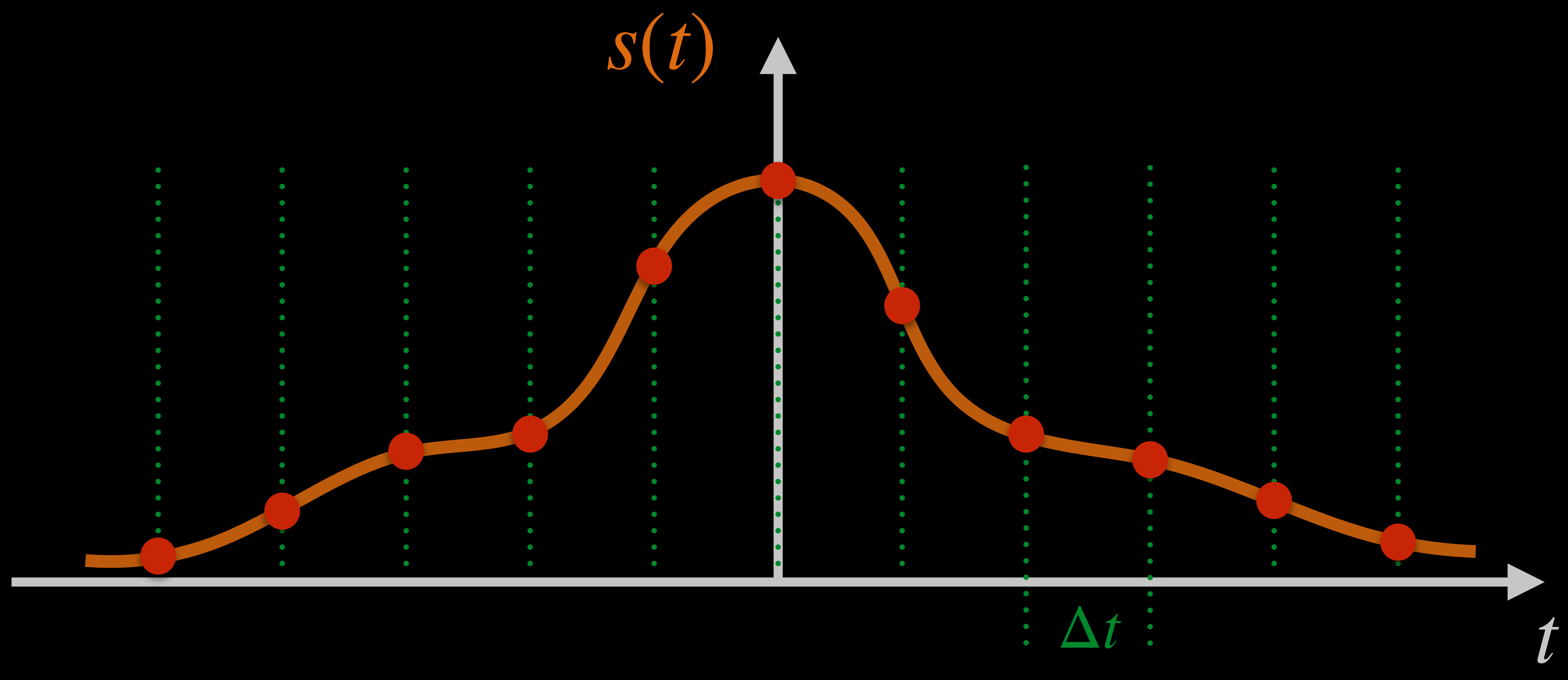
SIGNAL SAMPLING

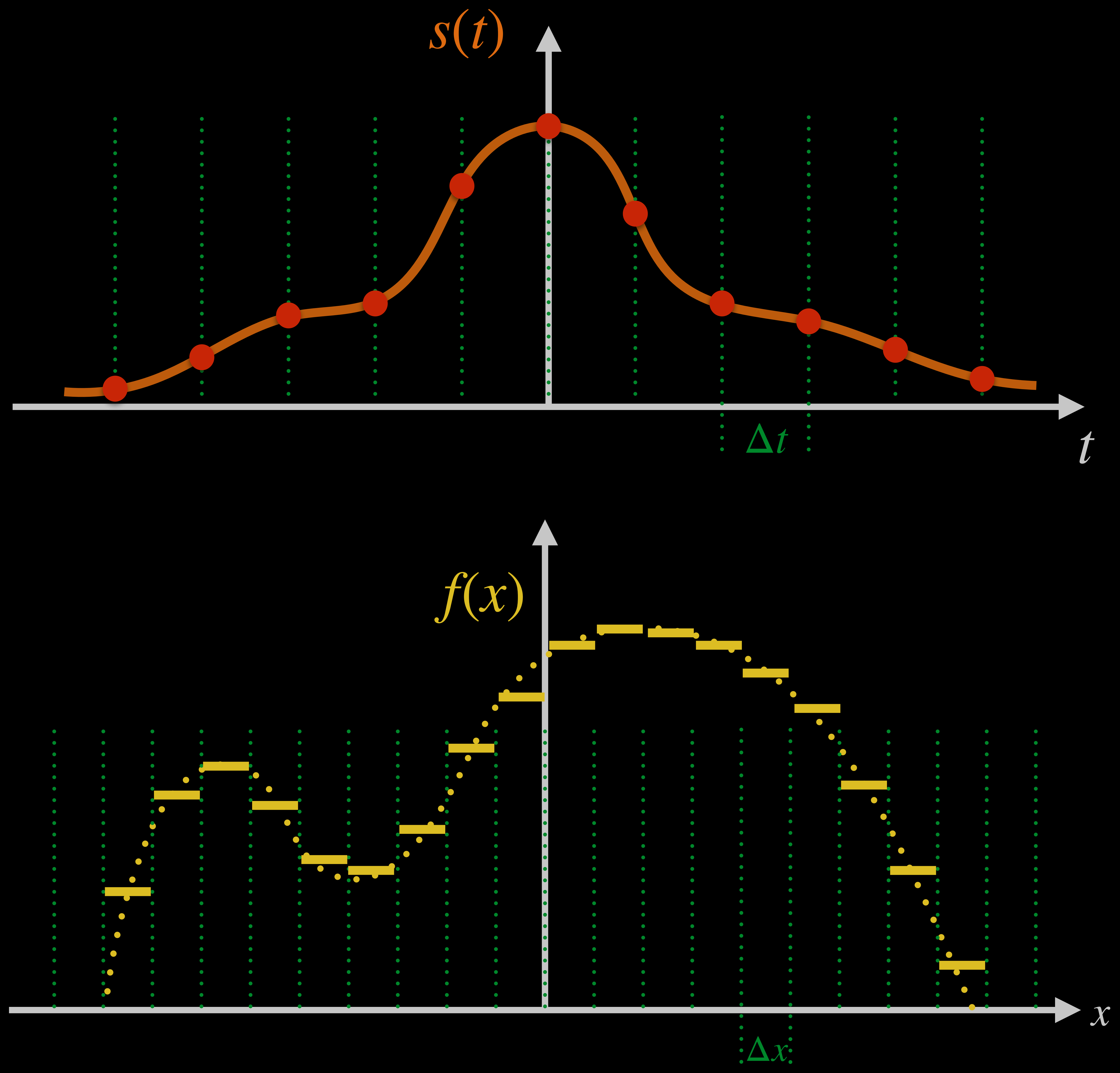


$$s(t) = \int f(x) e^{-i2\pi k_x(t)x} dx$$

$$s(t_q) = \int f(x) e^{-i2\pi k_x(t_q)x} dx$$

$$, t_q = q\Delta t$$

SIGNAL SAMPLING



$$s(t) = \int f(x) e^{-i2\pi k_x(t)x} dx$$

$$s(t_q) = \int f(x) e^{-i2\pi k_x(t_q)x} dx$$

$$, t_q = q\Delta t$$

$$s(q) = \sum_{n=1}^{N} f(x_n) e^{-i2\pi k_x(q)x_n}$$

$$, x_n = n\Delta x$$

ENCODING MODEL

$$y(q) = s(q) + \epsilon_q = \sum_{n=1}^{N} e^{-i2\pi k_x(q)n} f(n) + \epsilon_q$$

$$\mathbf{y} = \mathbf{Af} + \epsilon$$

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{f}} ||\mathbf{y} - \mathbf{Af}||^2$$

ENCODING MODEL

$$y(q) = s(q) + \epsilon_q = \sum_{n=1}^{N} e^{-i2\pi k_x(q)n} f(n) + \epsilon_q$$

$$\mathbf{y} = \mathbf{Af} + \epsilon$$

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{f}} ||\mathbf{y} - \mathbf{Af}||^2 = \mathbf{A}^{-1}\mathbf{y} = DFT^{-1}\{\mathbf{y}\}$$

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \psi & \psi^2 & \dots & \psi^{N-1} \\ 1 & \psi^2 & \psi^4 & \dots & \psi^{N-2} \\ \vdots & & & & \\ 1 & \psi^{N-1} & \psi^{N-2} & \dots & \psi \end{bmatrix}$$

$$\psi = e^{-i2\pi nq}$$

ENCODING MODEL

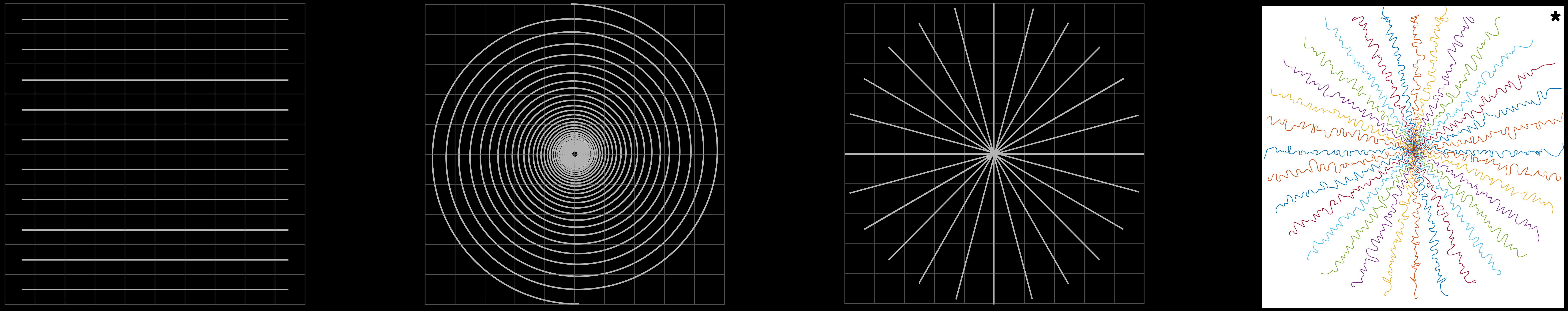
$$y(q) = s(q) + \epsilon_q = \sum_{n=1}^{N} e^{-i2\pi k_x(q)n} f(n) + \epsilon_q$$

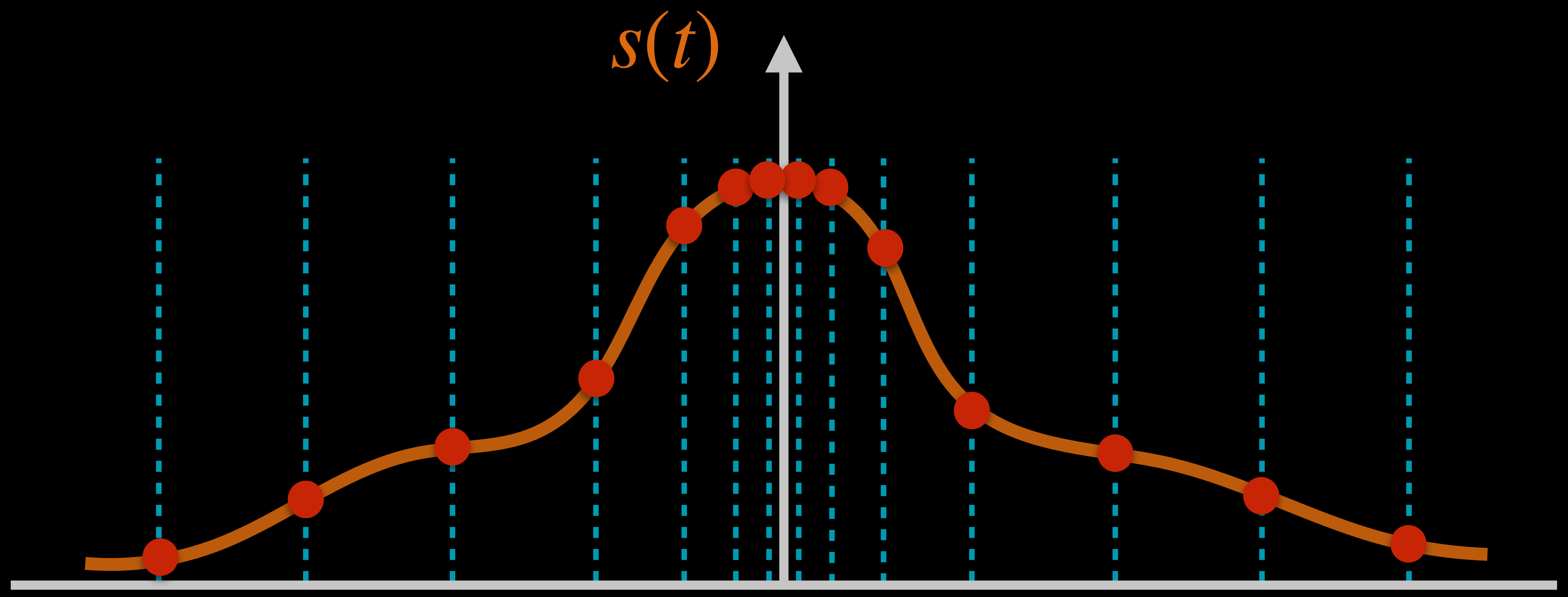
$$\mathbf{y} = \mathbf{A}\mathbf{f} + \epsilon$$

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{f}} ||\mathbf{y} - \mathbf{A}\mathbf{f}||^2 \ + \lambda \mathbf{R}(\mathbf{y})$$

- non-cartesian sampling
- field inhomogeneity
- signal decay
- coil sensitivities
- …

NON-CARTESIAN SAMPLING

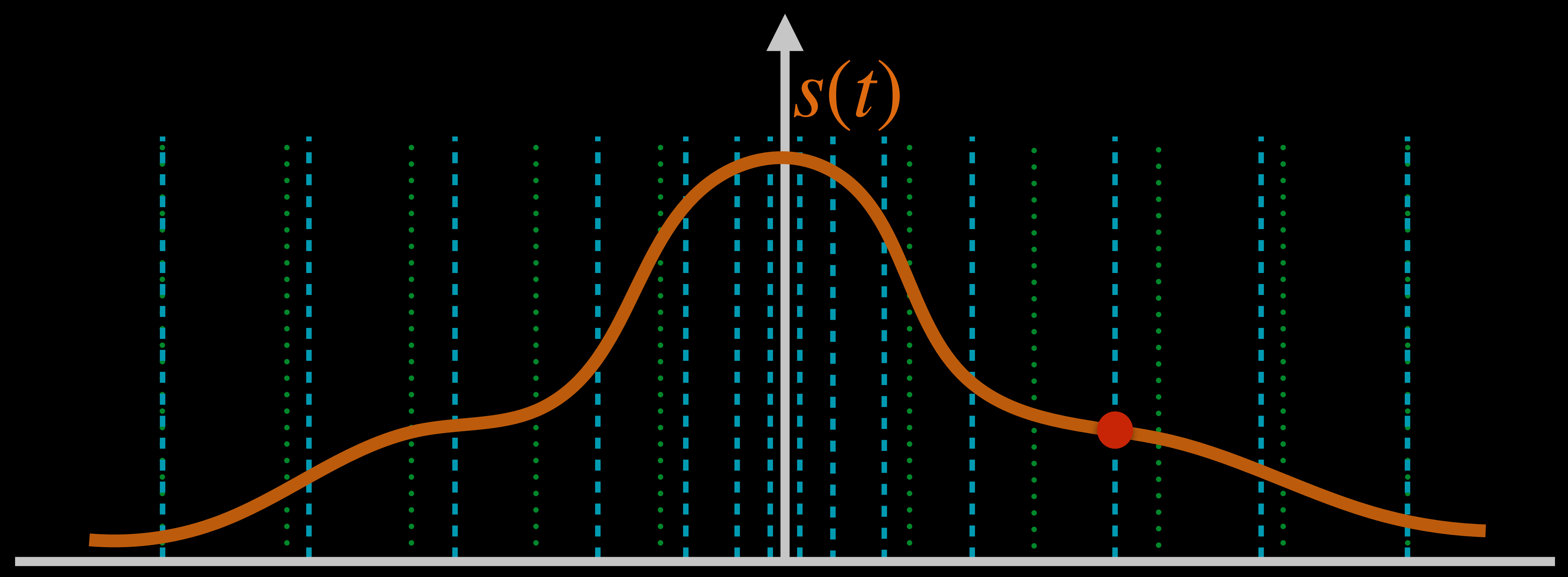




Non-**U**nifrom **FFT**
(gridding + FFT)

* G. Wang et al, ISMRM 2021

GRIDDING



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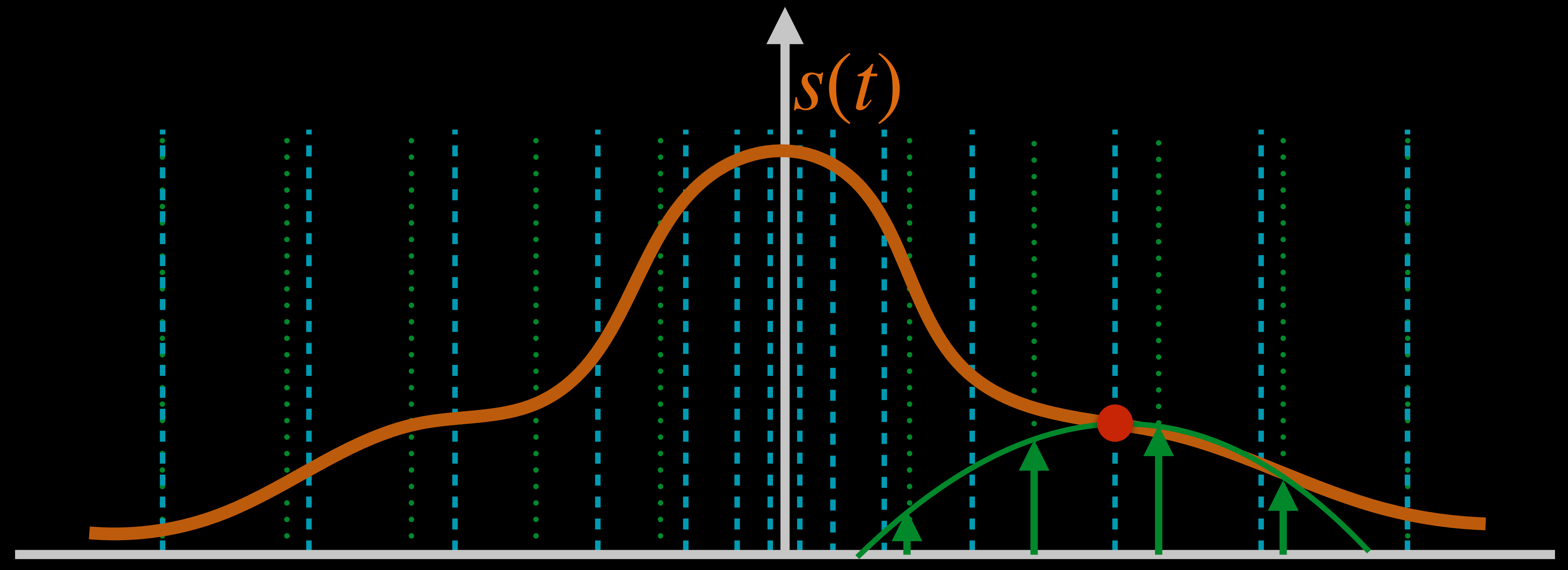

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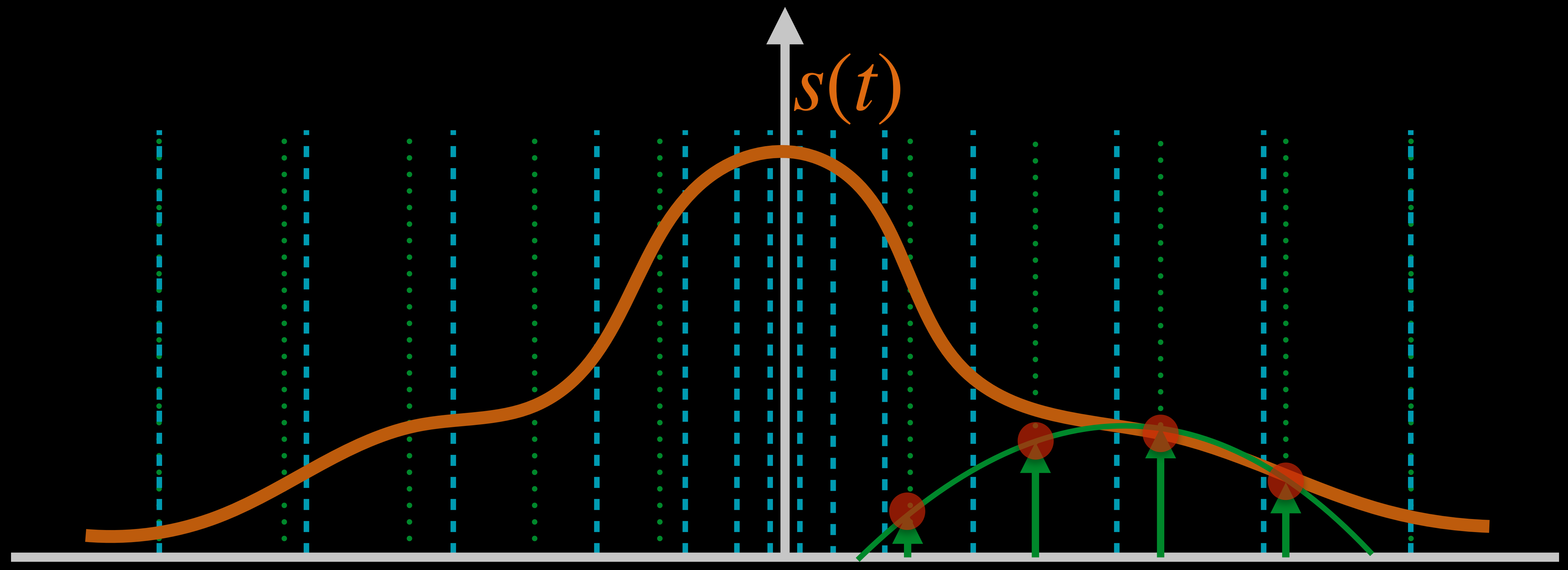

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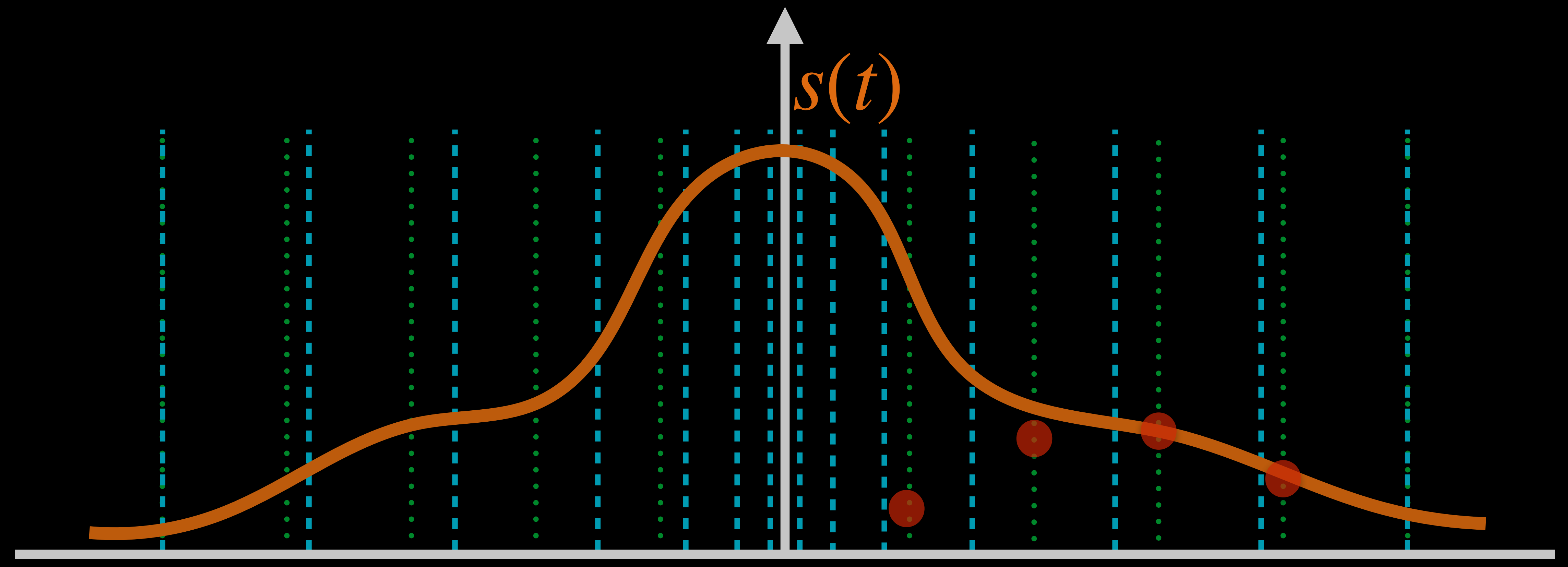

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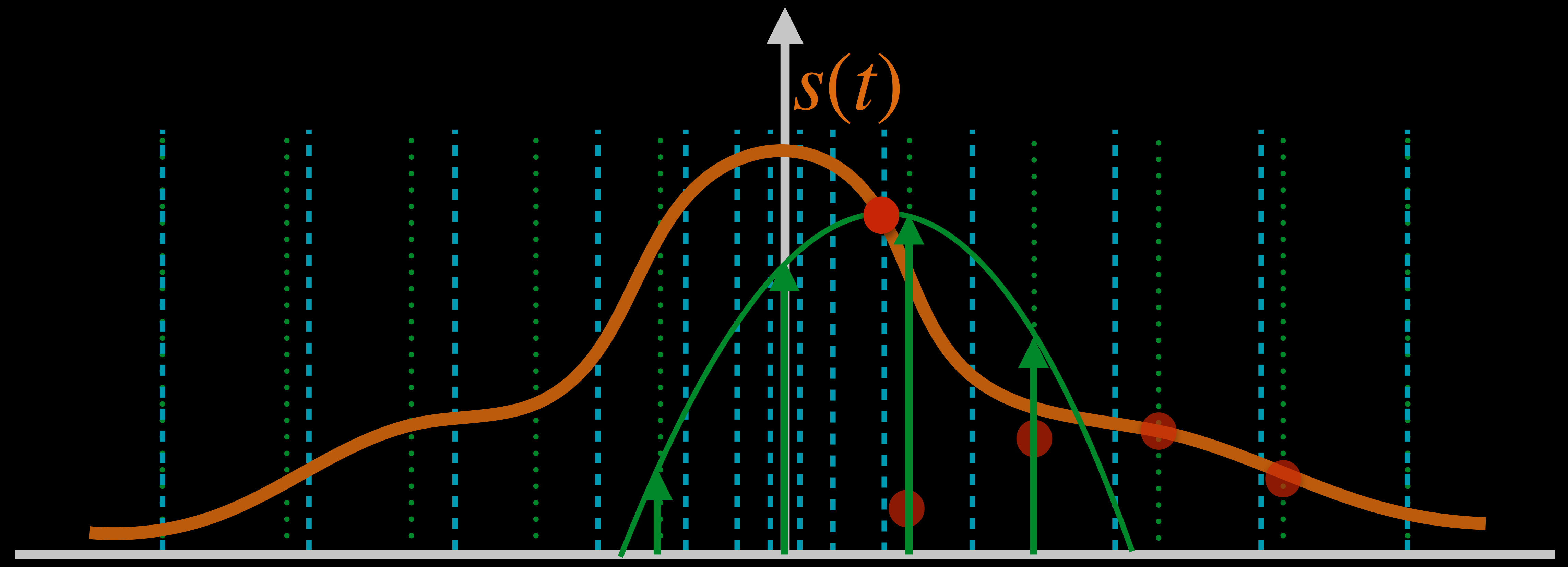

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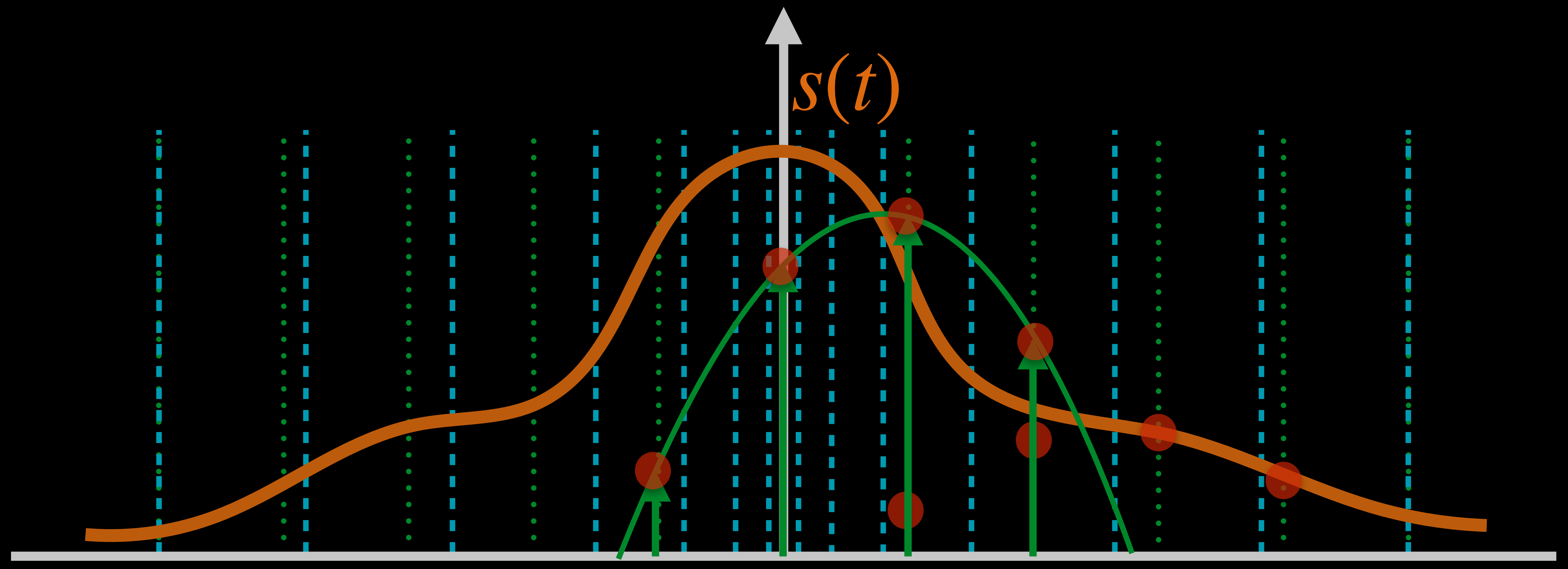

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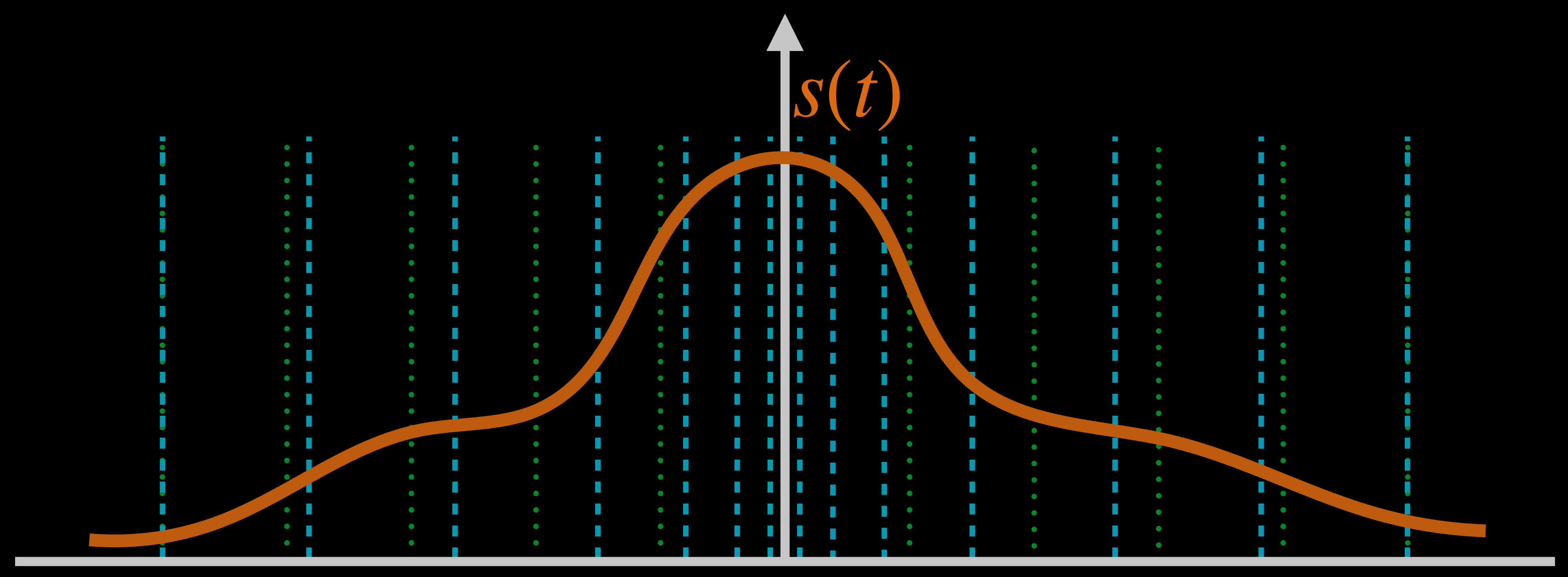

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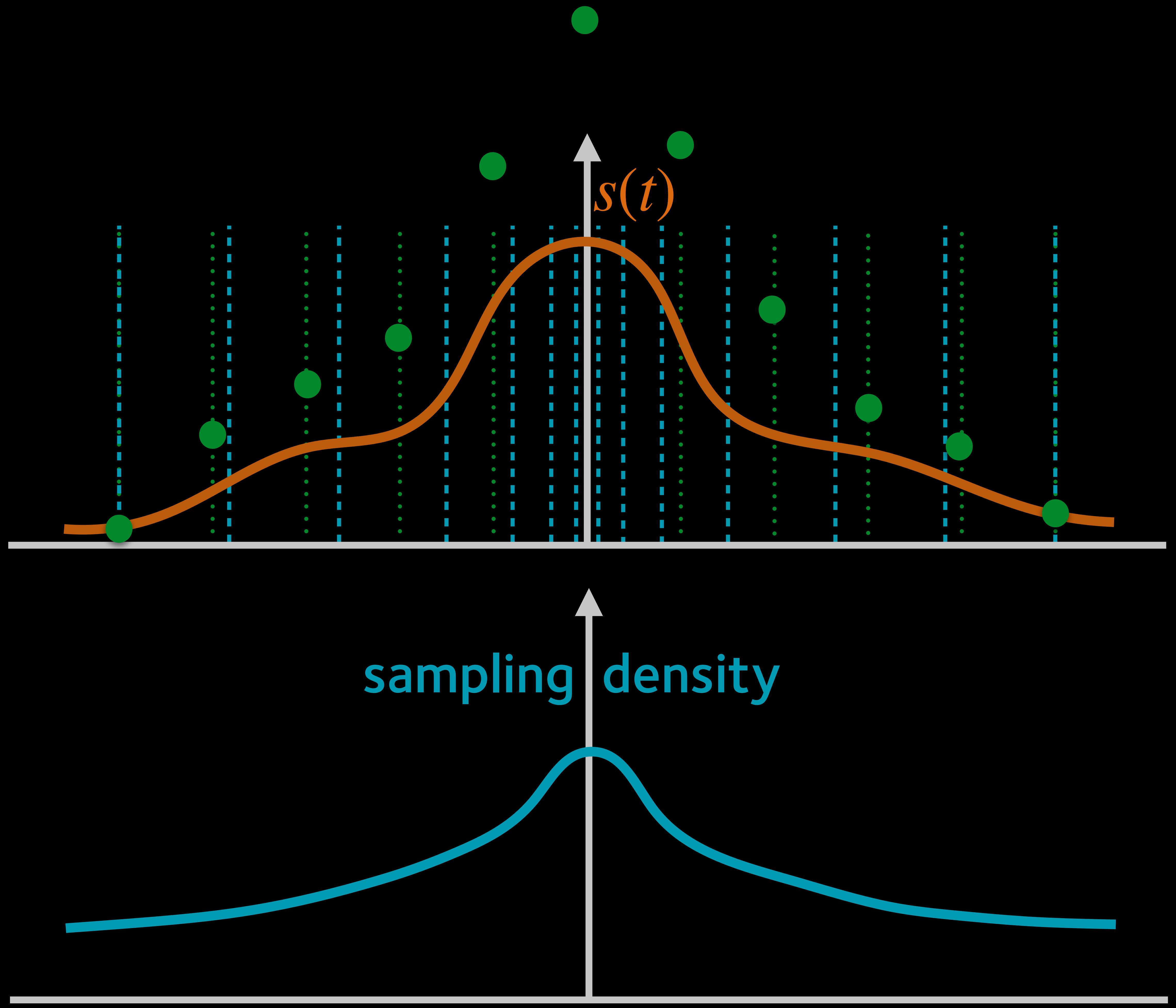

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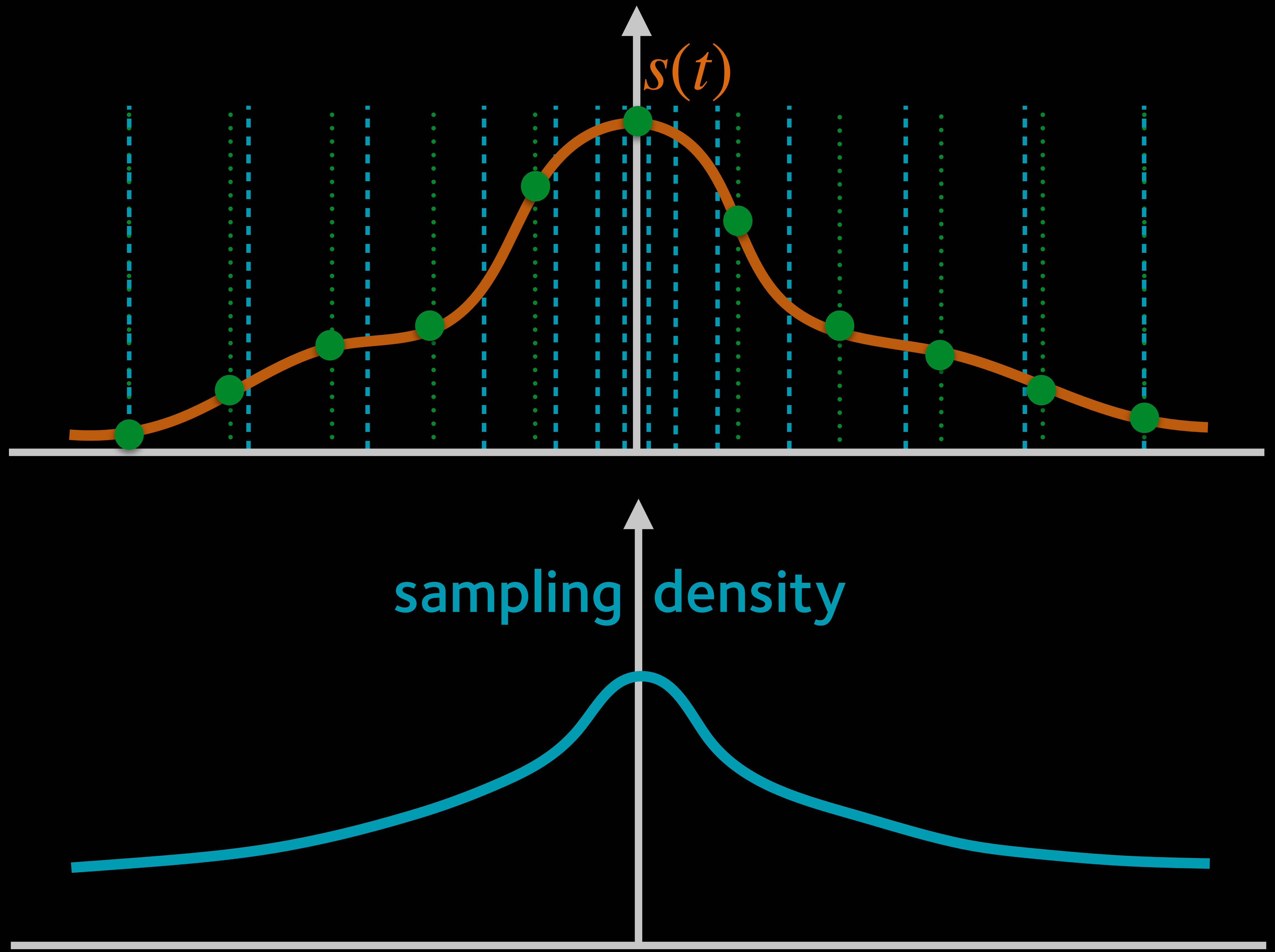

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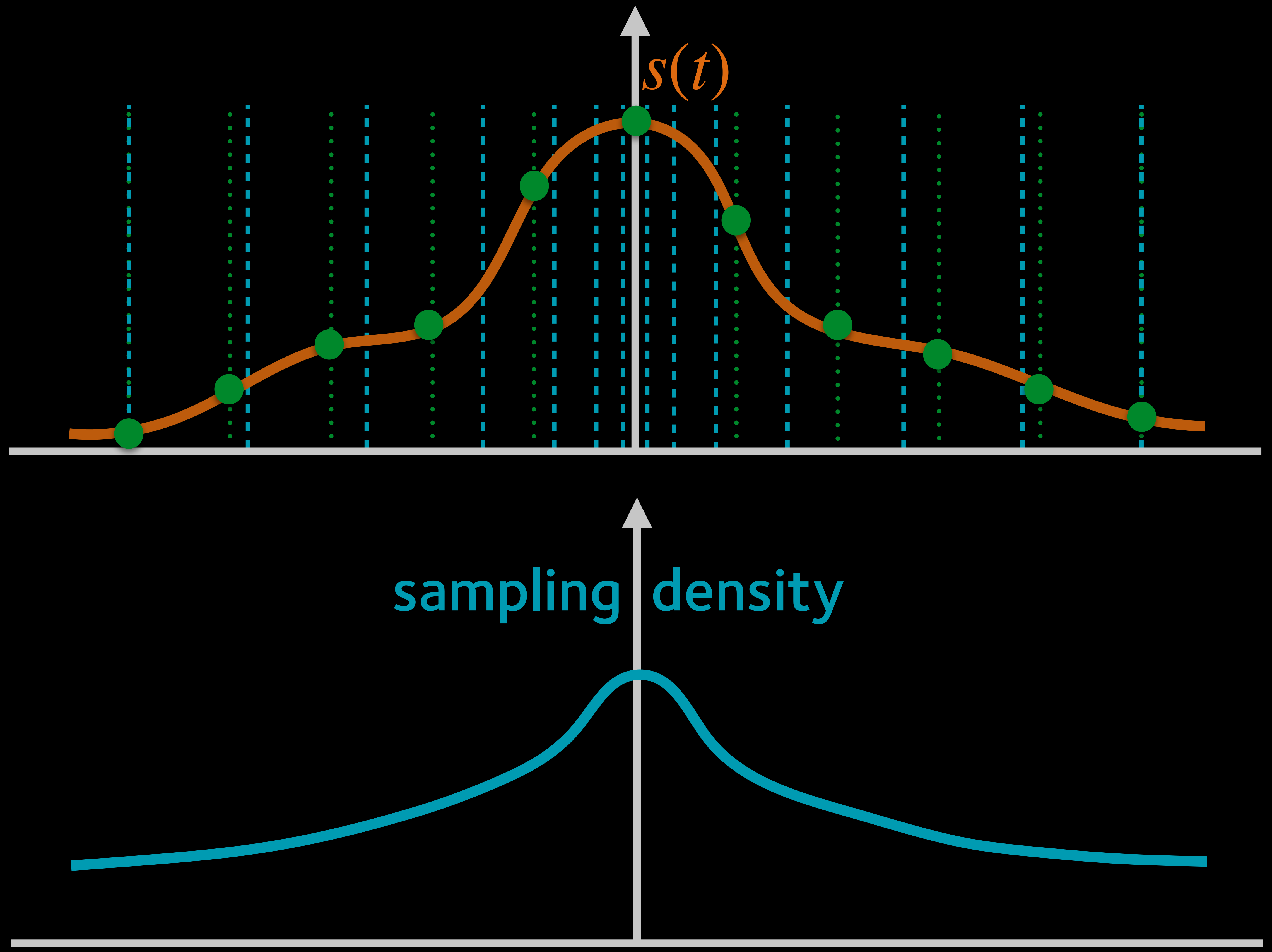
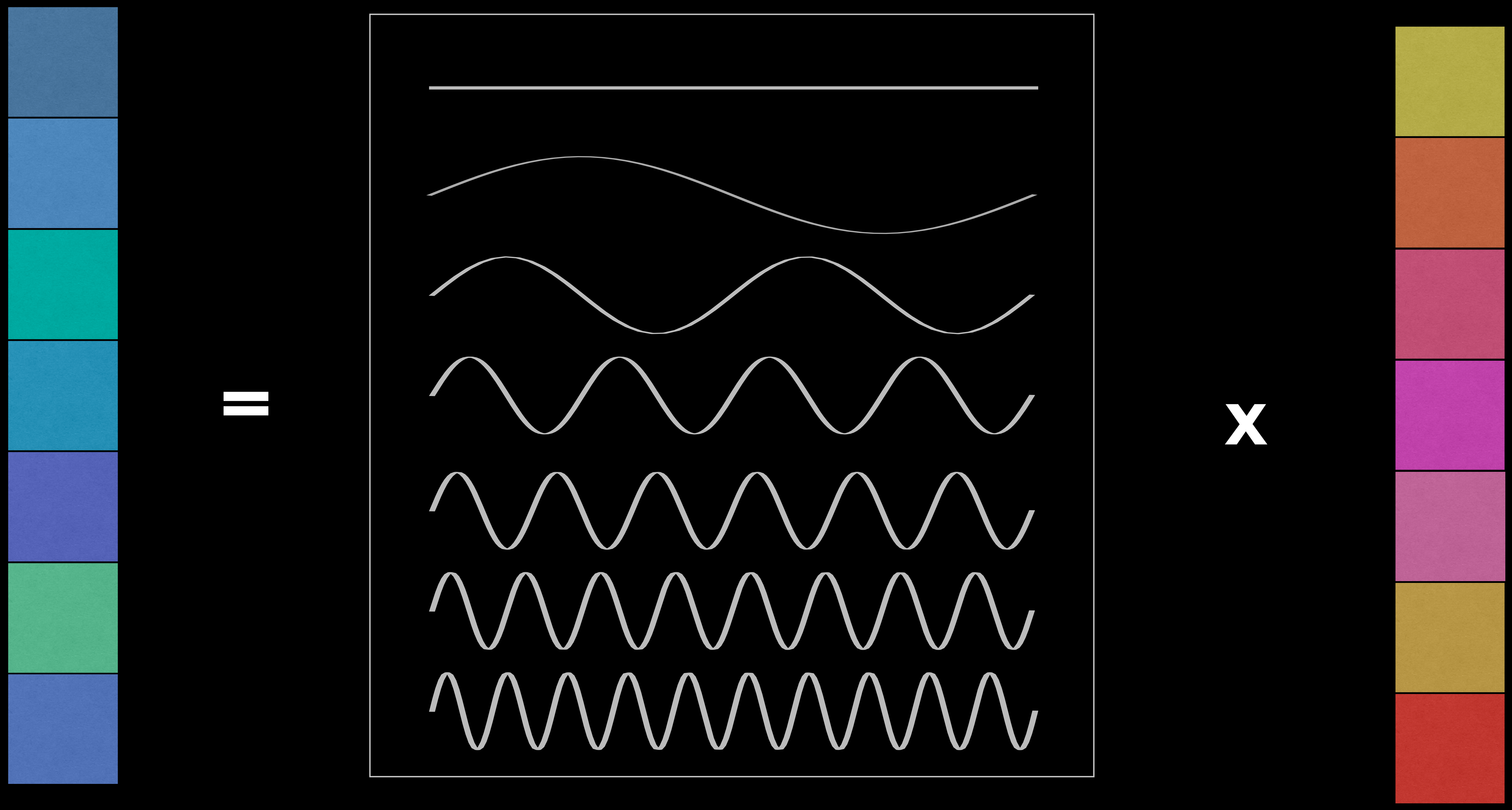


practical considerations:

- grid resolution
- kernel function and specs
- density estimation

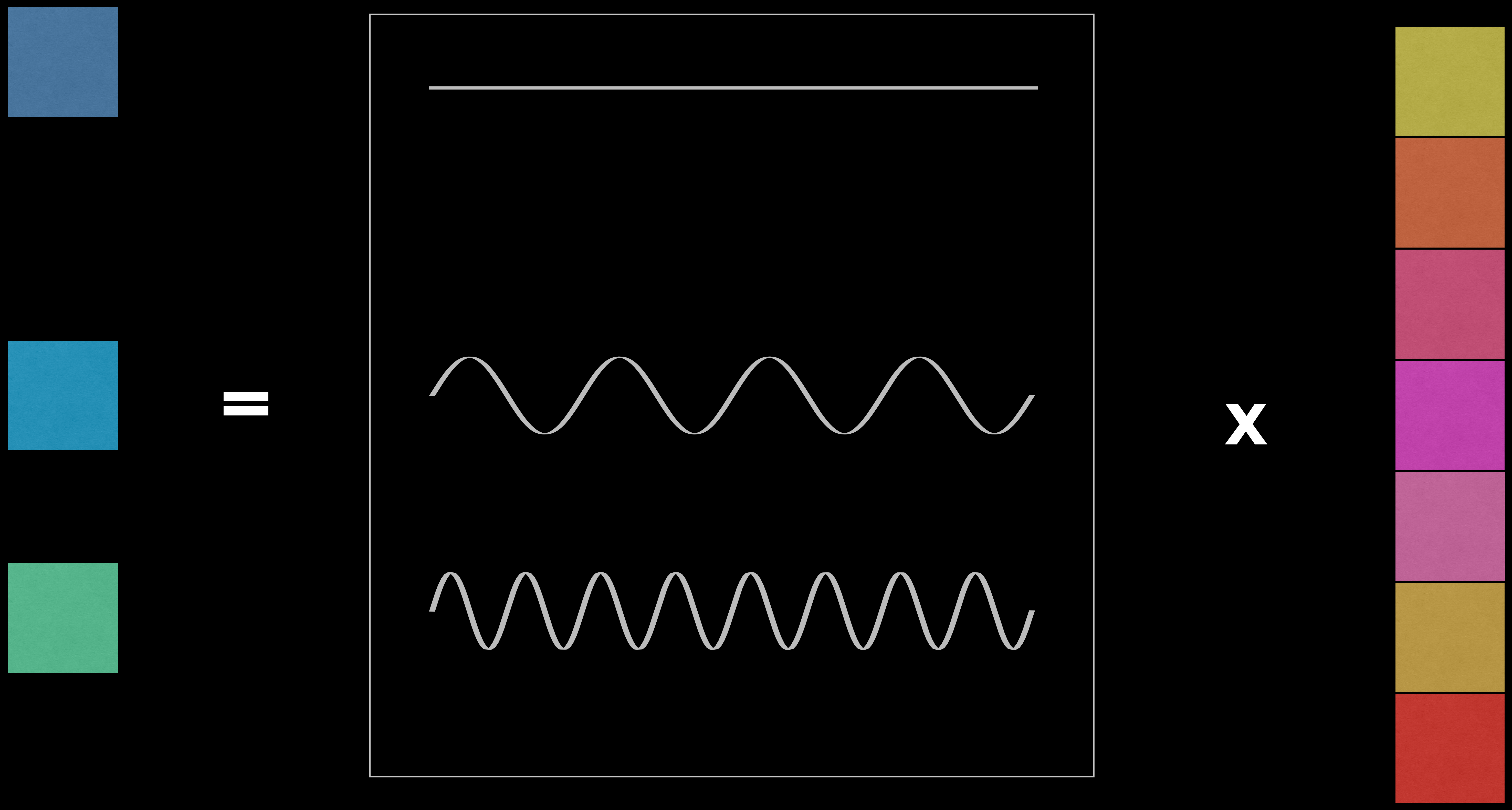
ACCELERATED MRI



y = A x f

$$\hat{f} = A^{-1}y$$

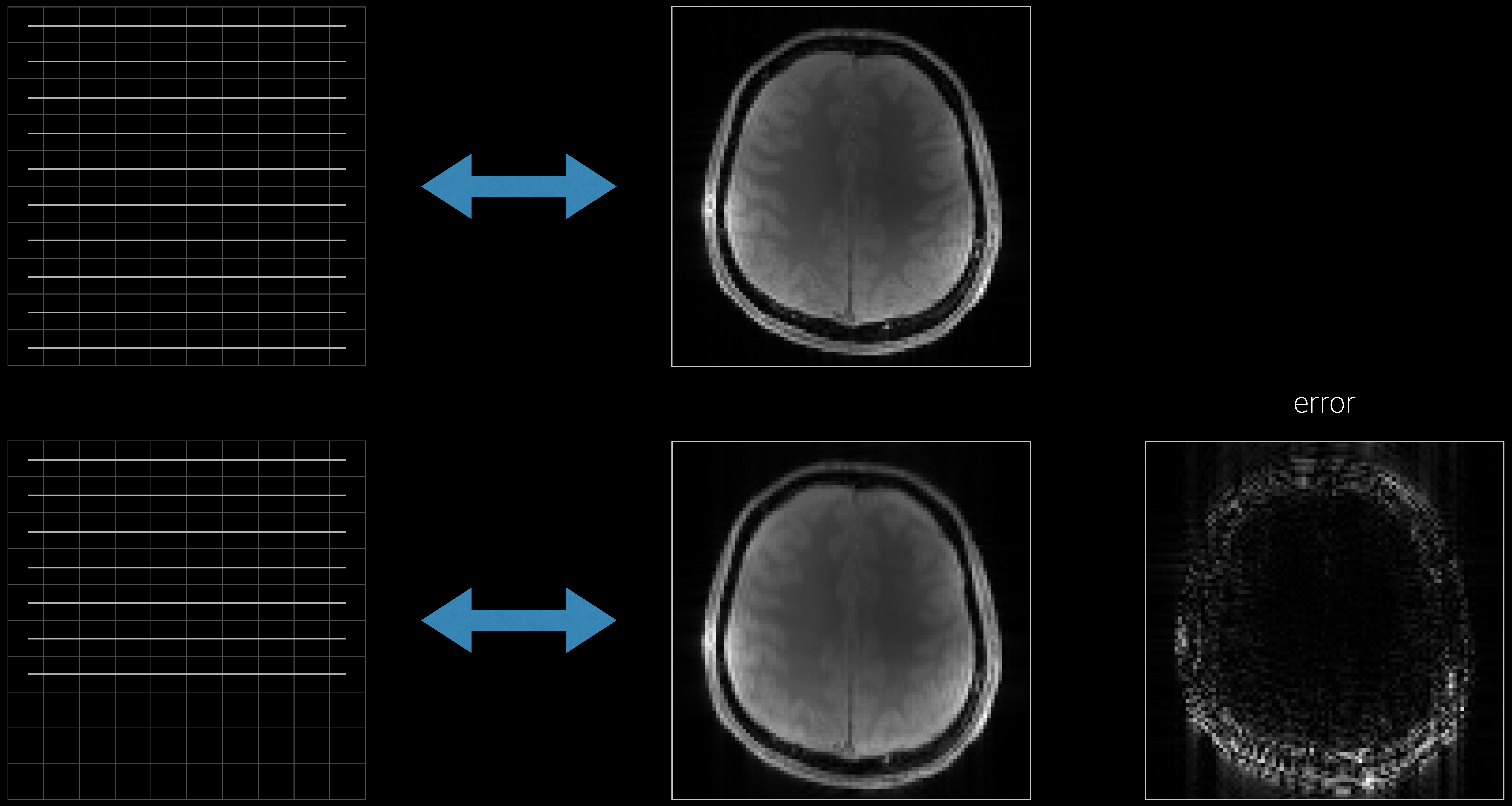
ACCELERATED MRI



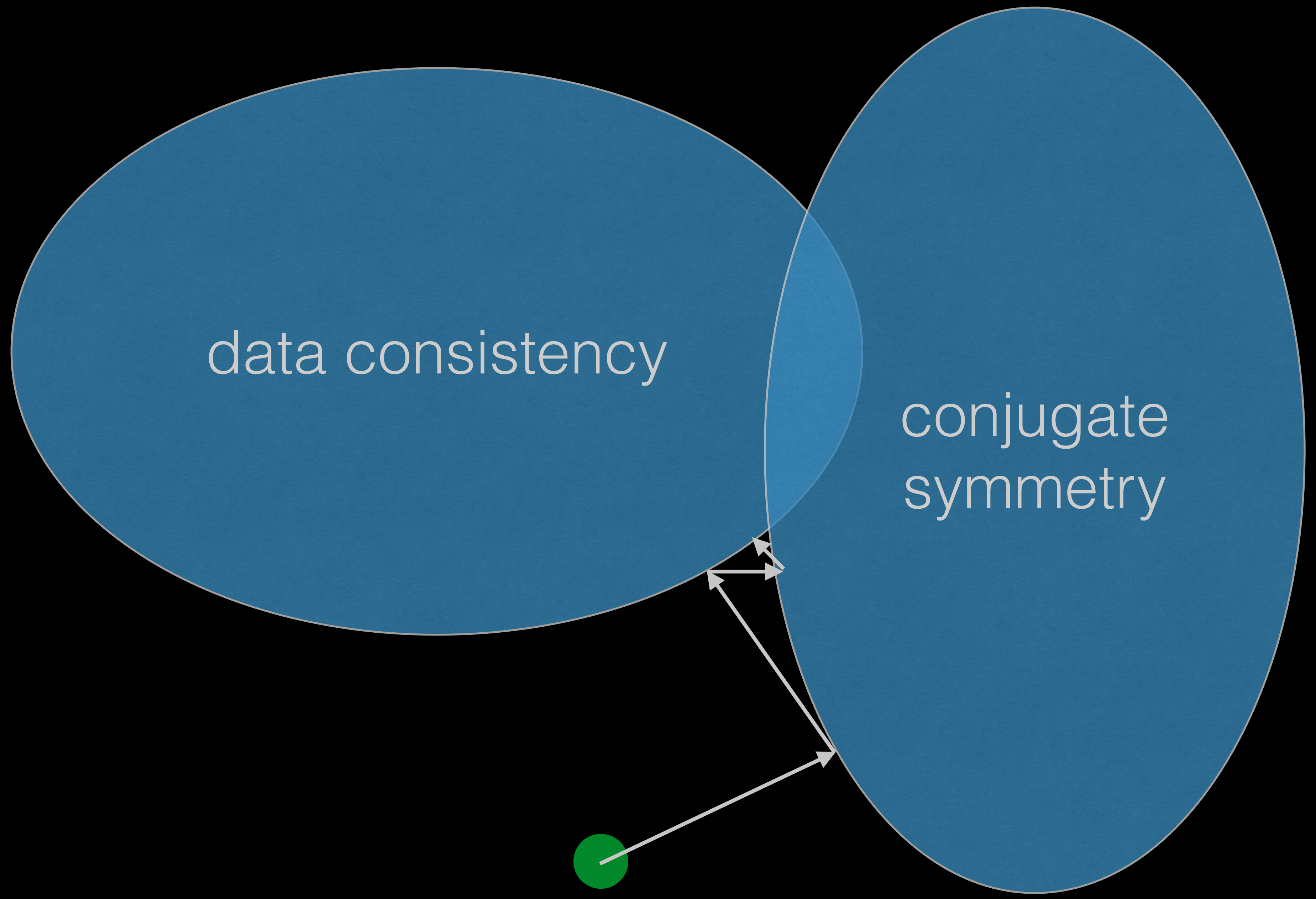
y = A x f

$$\hat{f} = A^{-1}y$$

ACCELERATED MRI: PARTIAL FOURIER

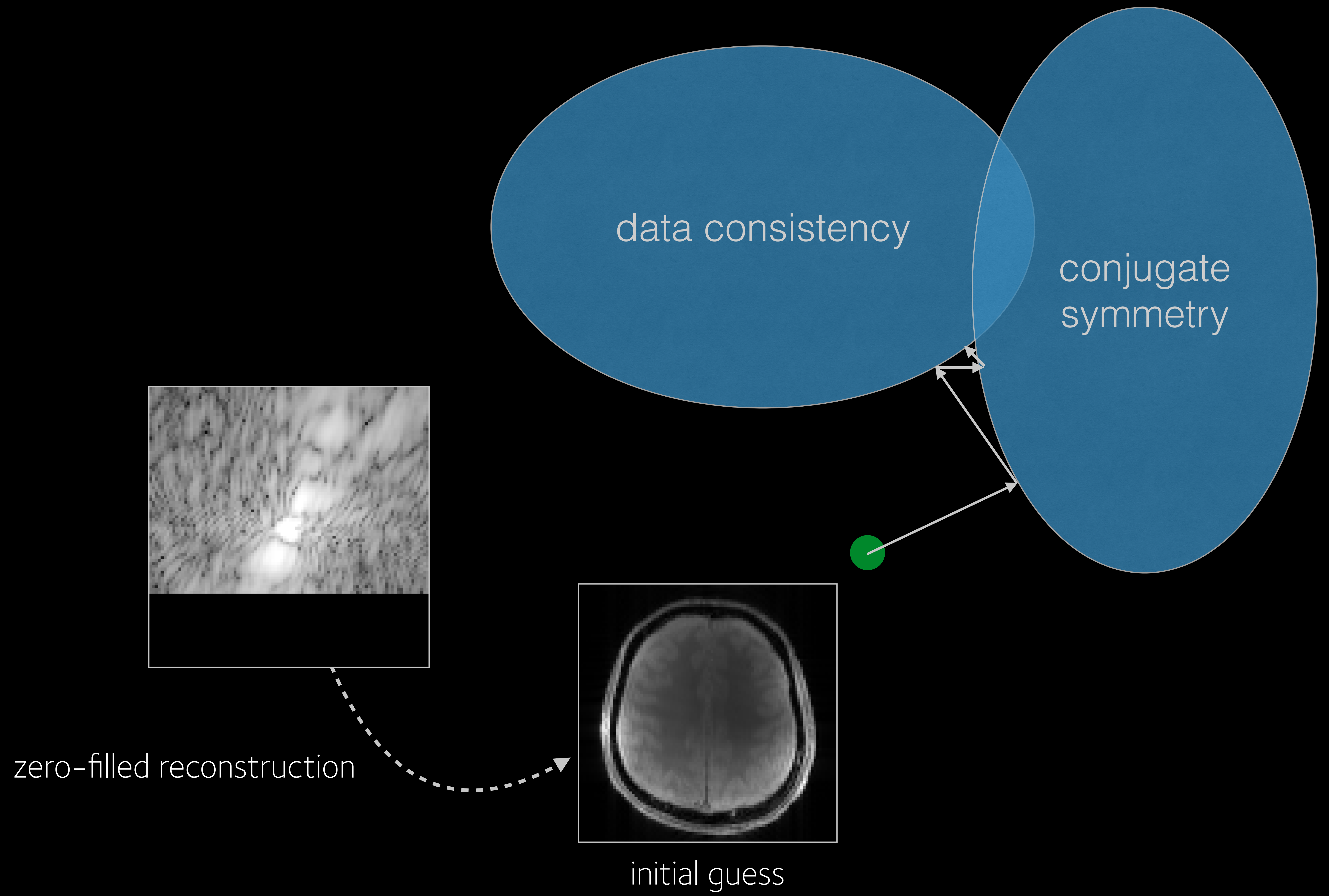


POCS RECONSTRUCTION

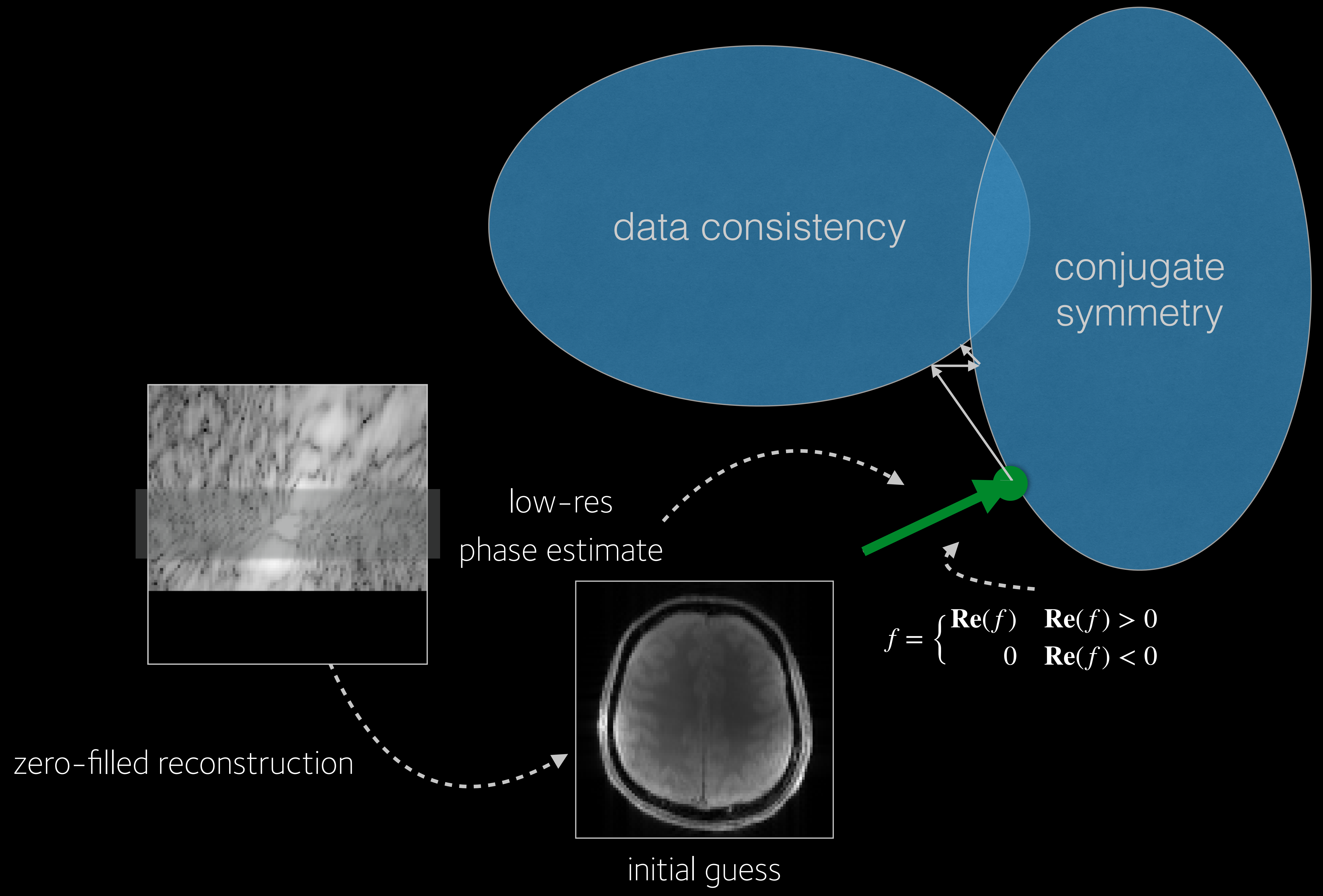


initial guess

POCS RECONSTRUCTION

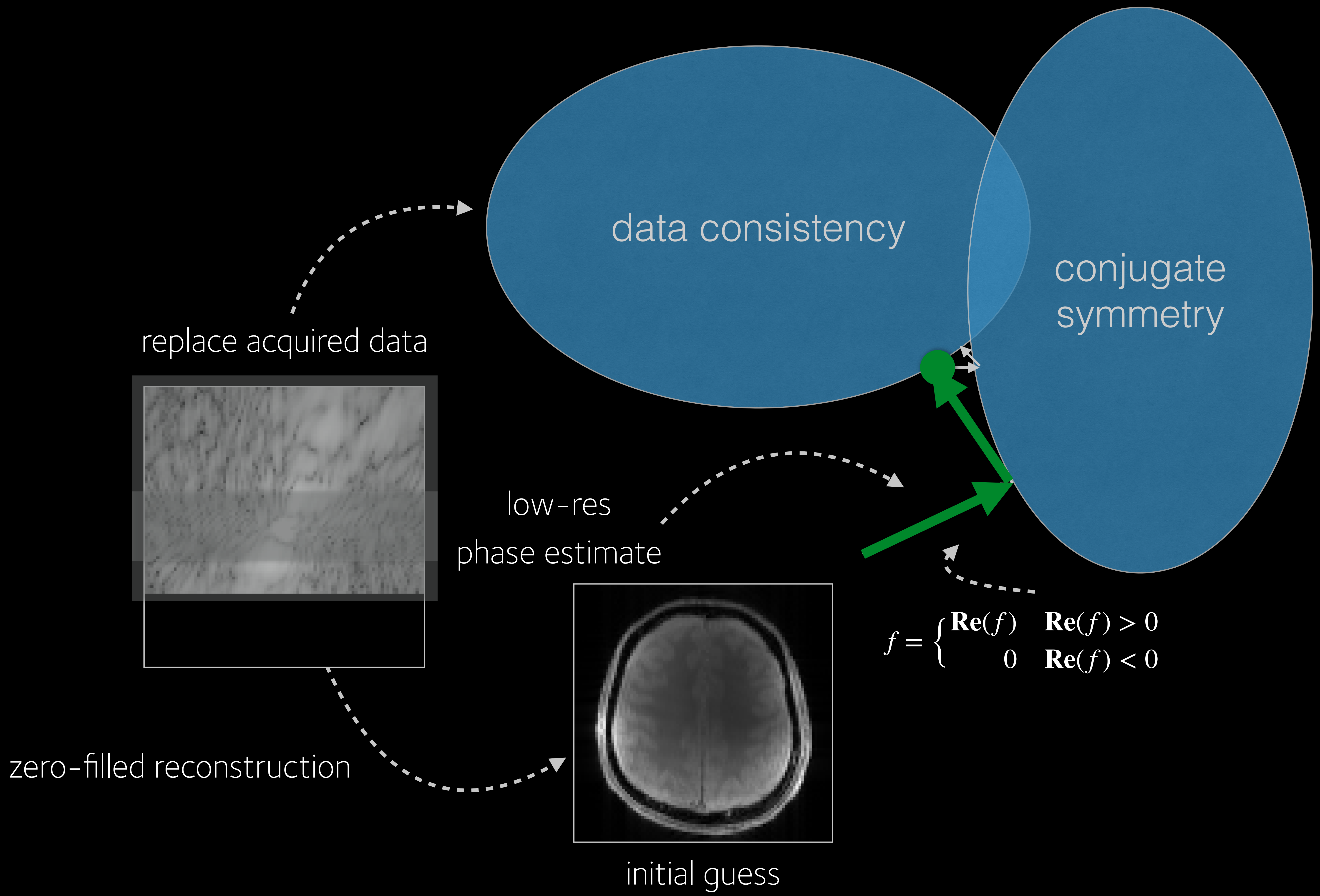


POCS RECONSTRUCTION



$$f = \begin{cases} \mathbf{Re}(f) & \mathbf{Re}(f) > 0 \\ 0 & \mathbf{Re}(f) < 0 \end{cases}$$

POCS RECONSTRUCTION



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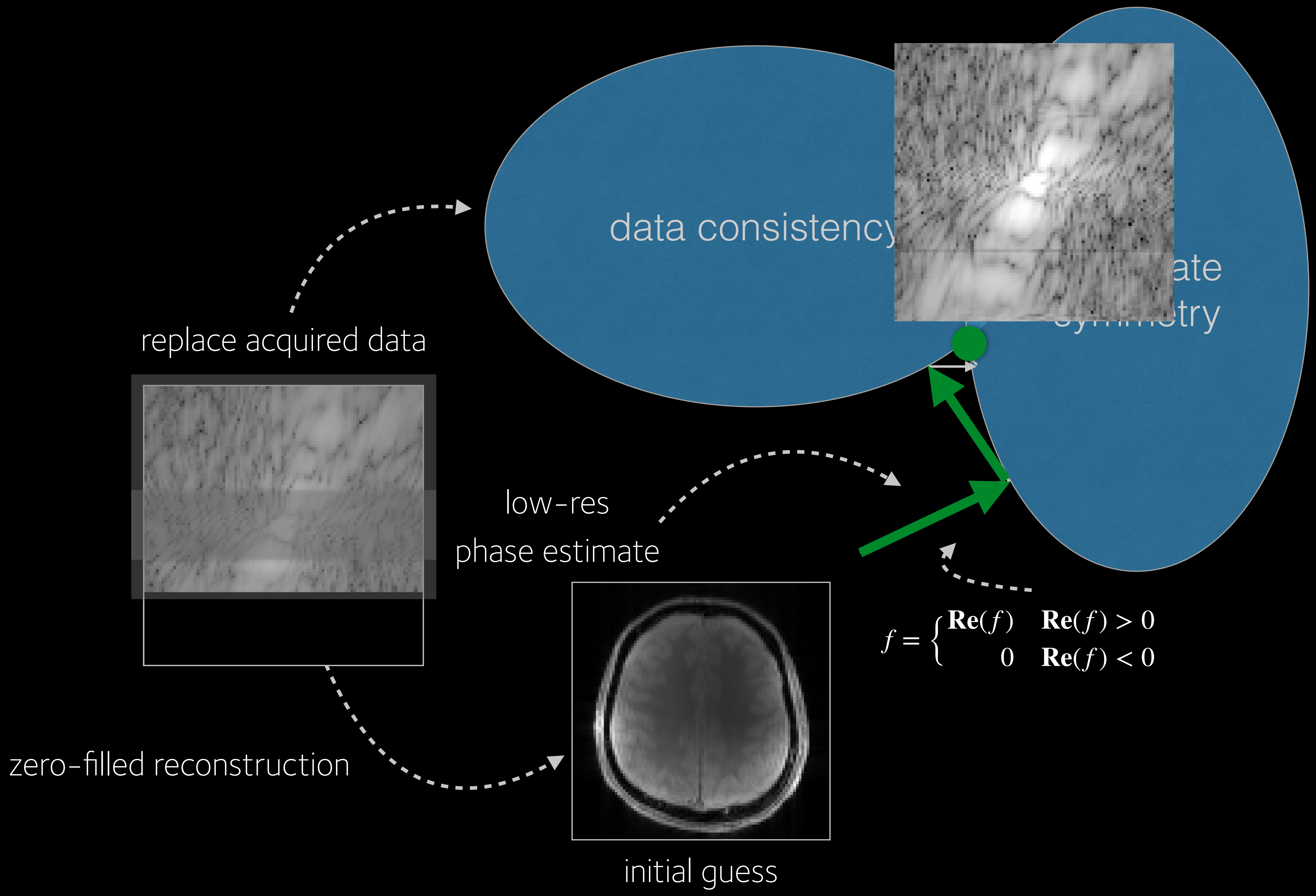
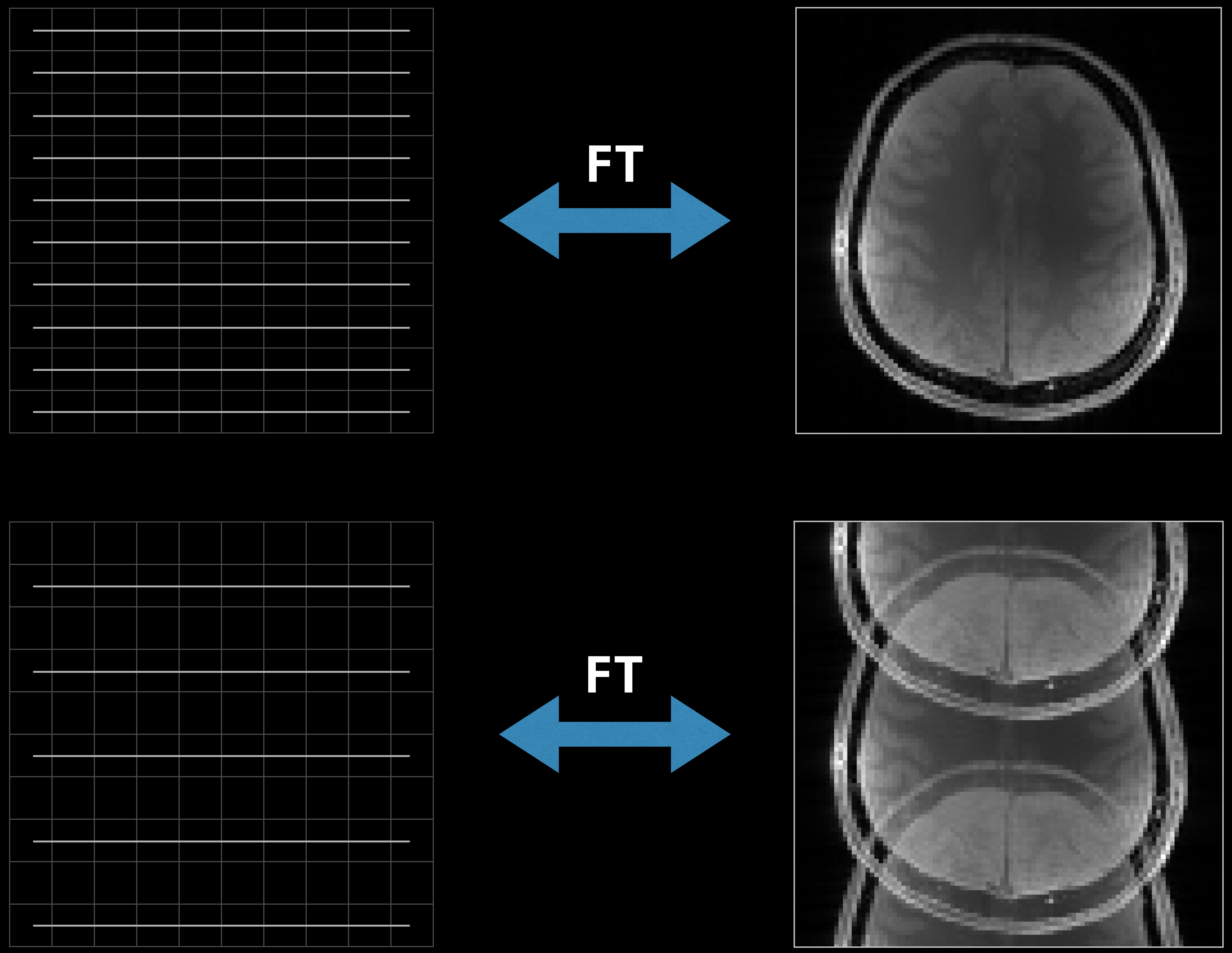
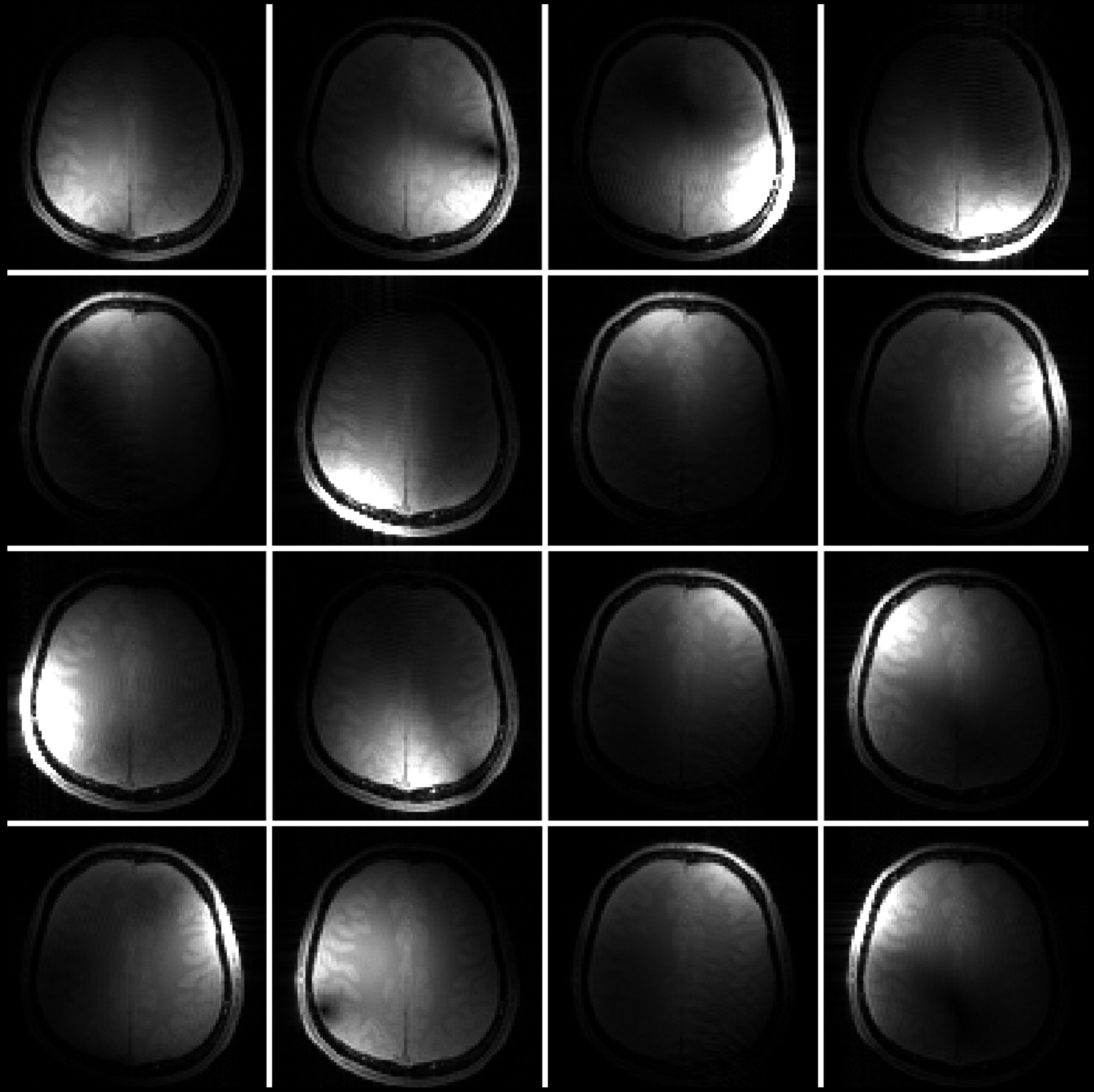


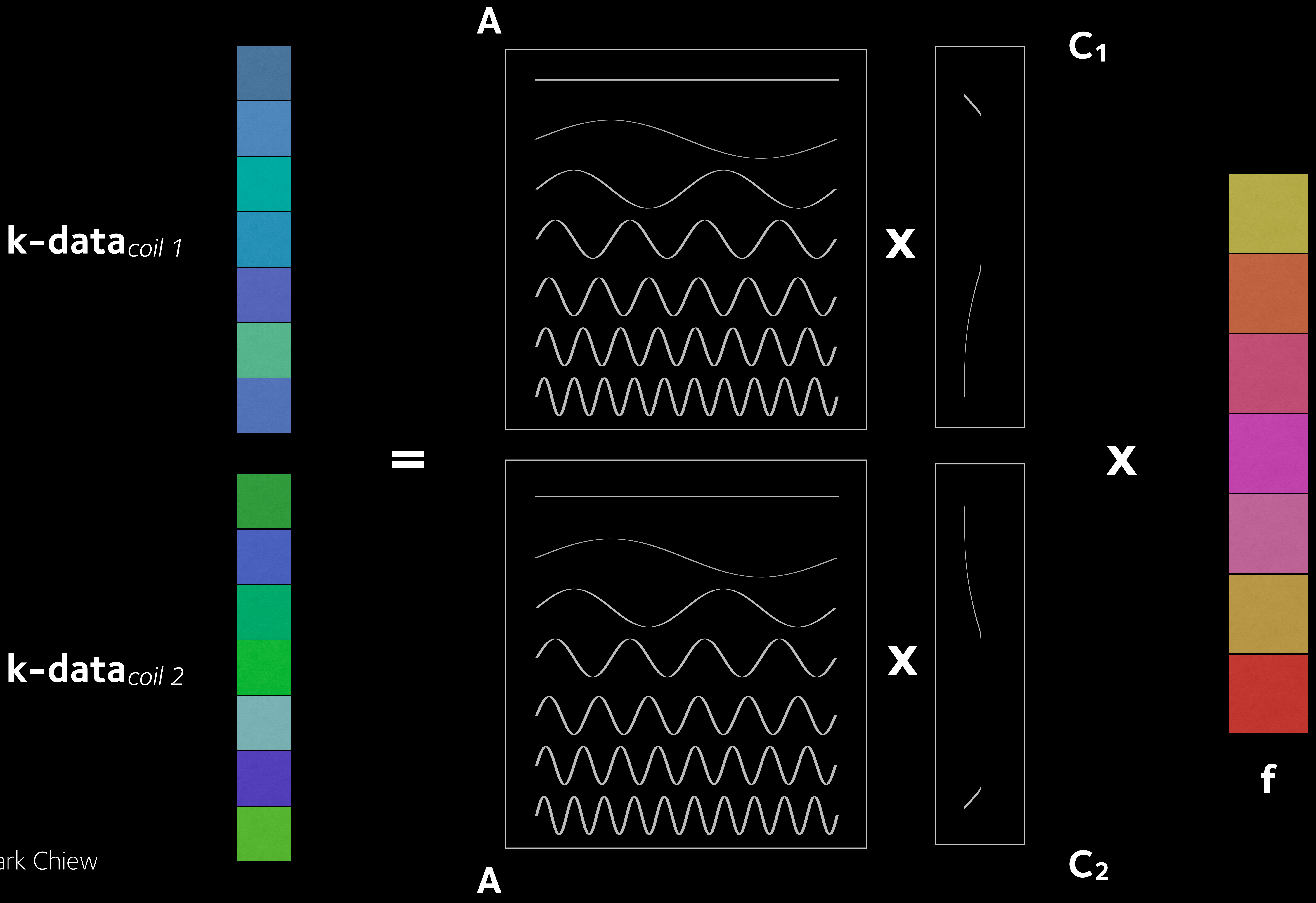
PARALLEL IMAGING



PHASED-ARRAY COILS

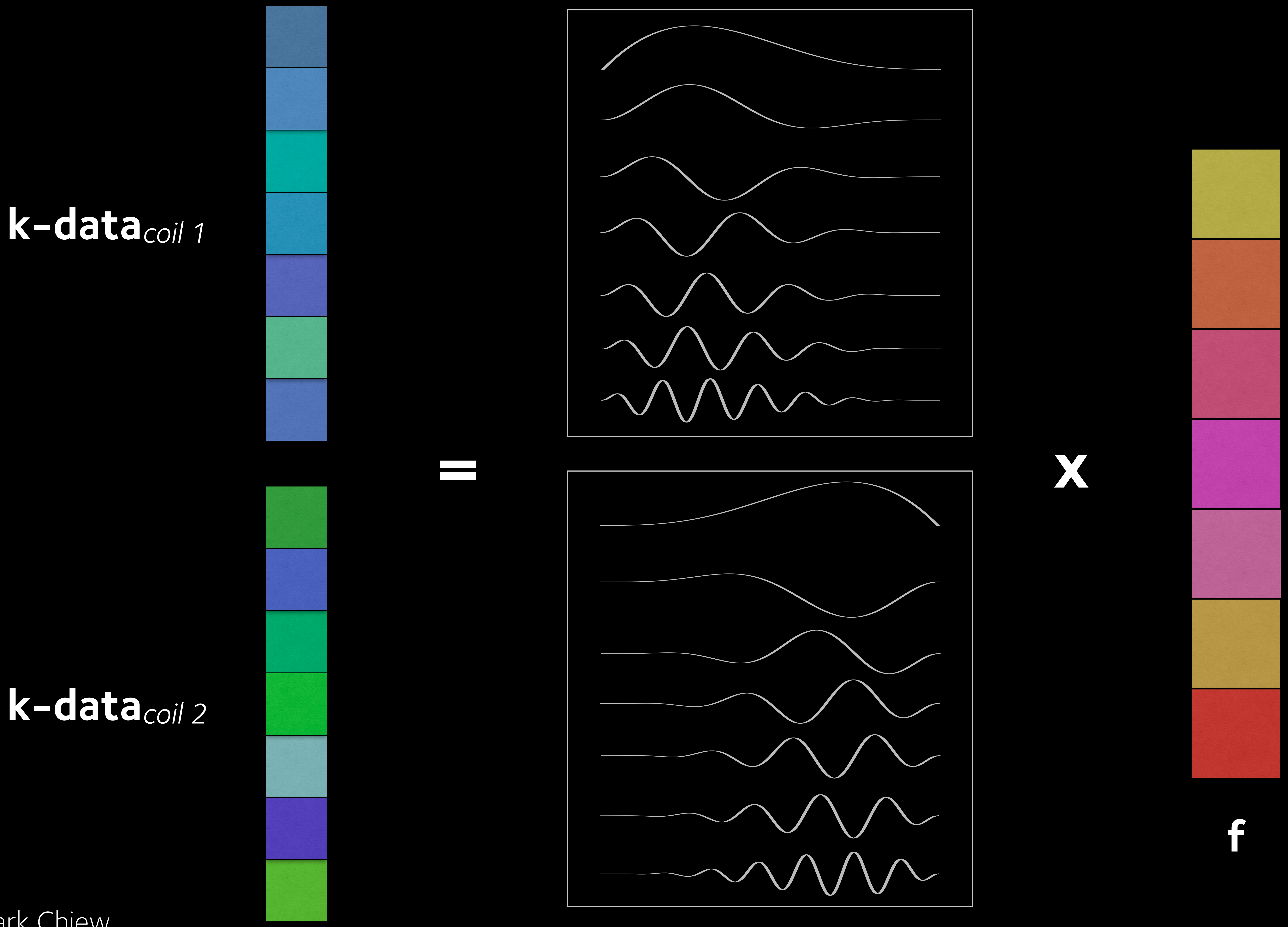


DATA REDUNDANCY



courtesy: Mark Chiew

DATA REDUNDANCY



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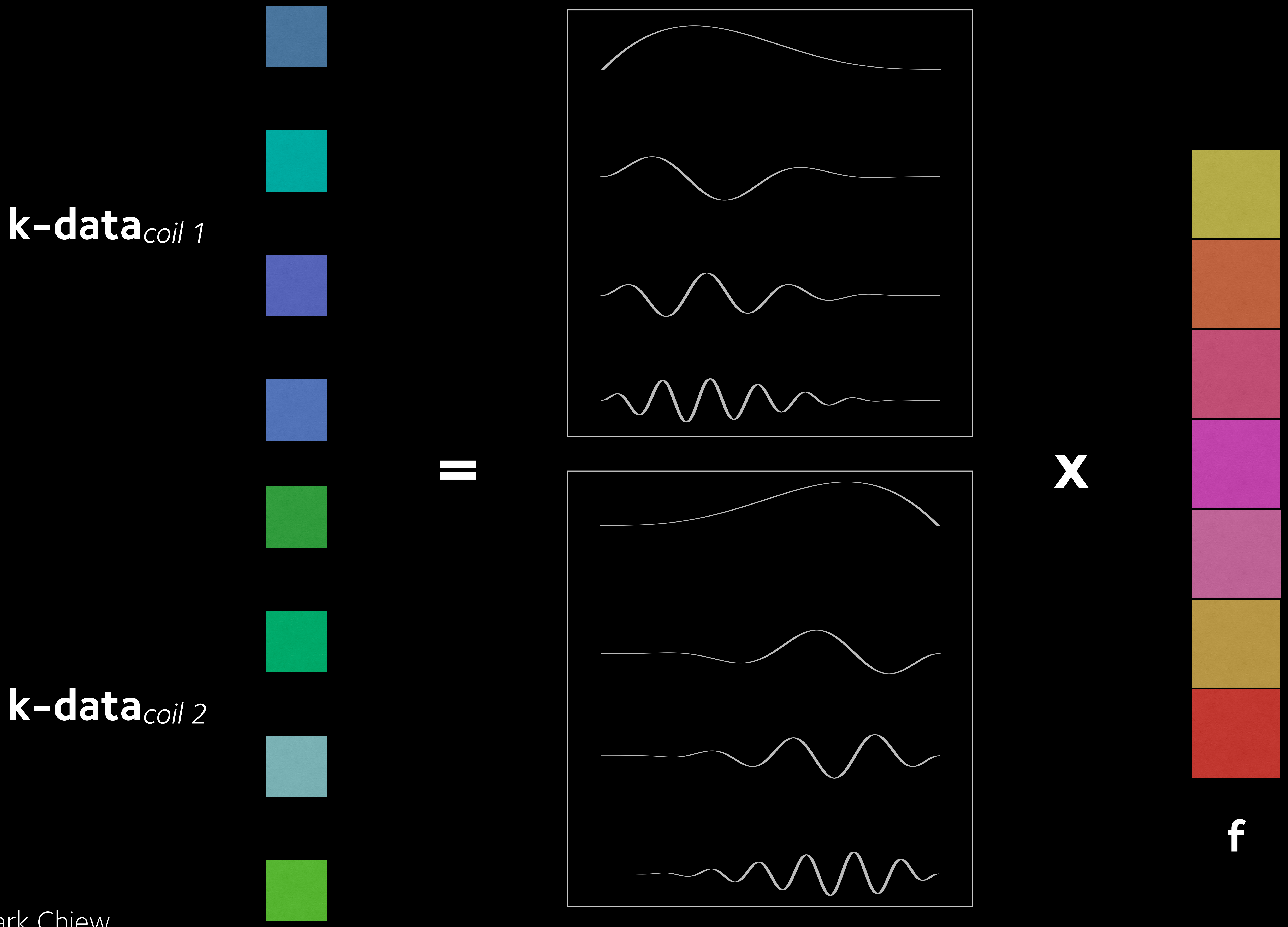
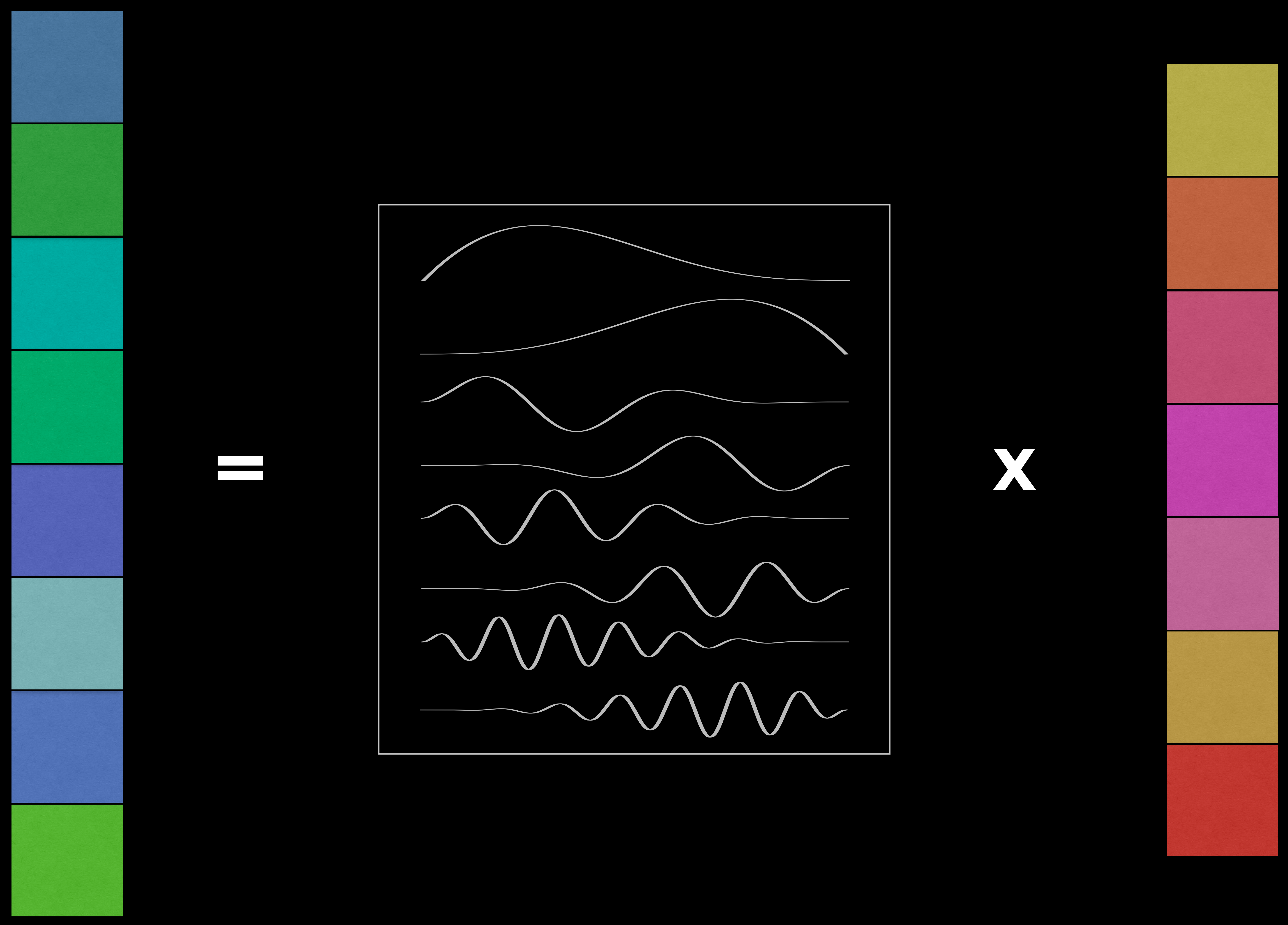


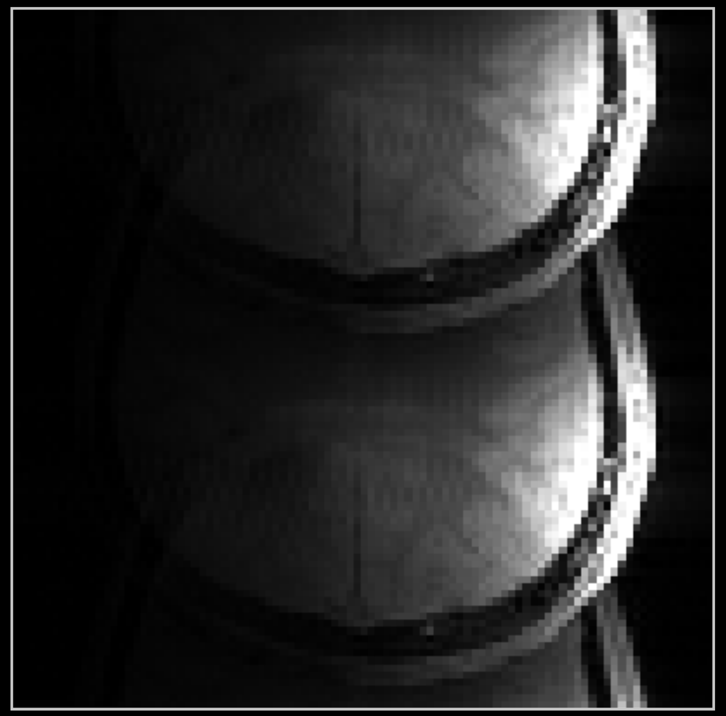
courtesy: Mark Chiew

DATA REDUNDANCY



SENSITIVITY ENCODING (SENSE)

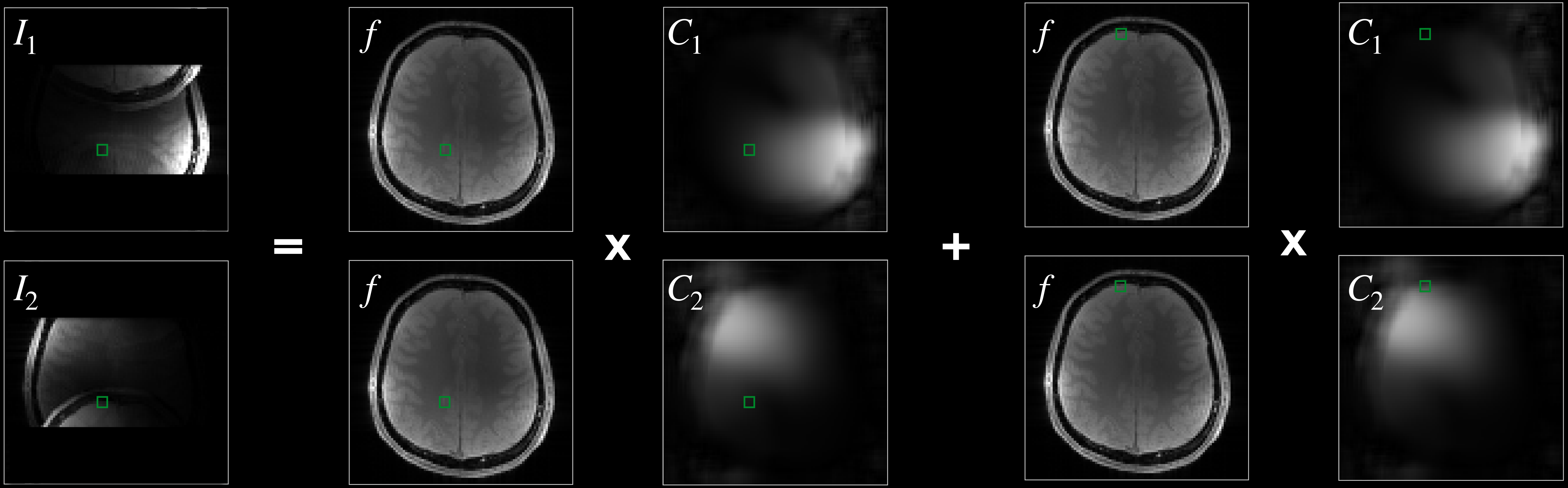
coil 1



coil 2

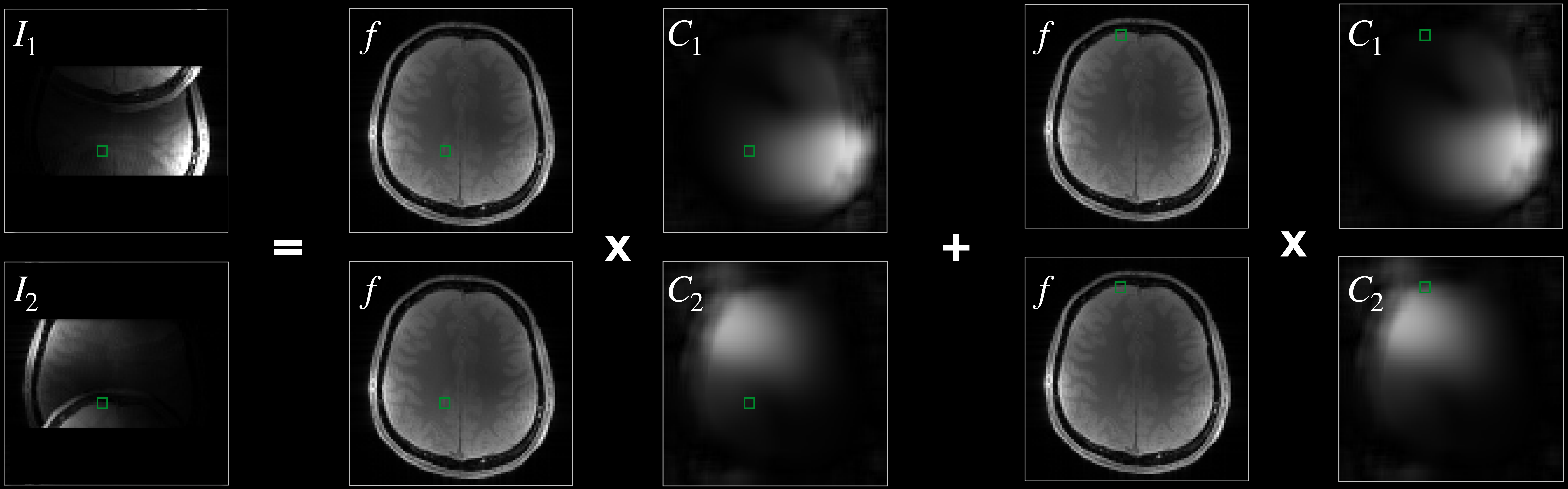
Pruessmann et al., 1999

SENSITIVITY ENCODING (SENSE)



Pruessmann et al., 1999

SENSITIVITY ENCODING (SENSE)



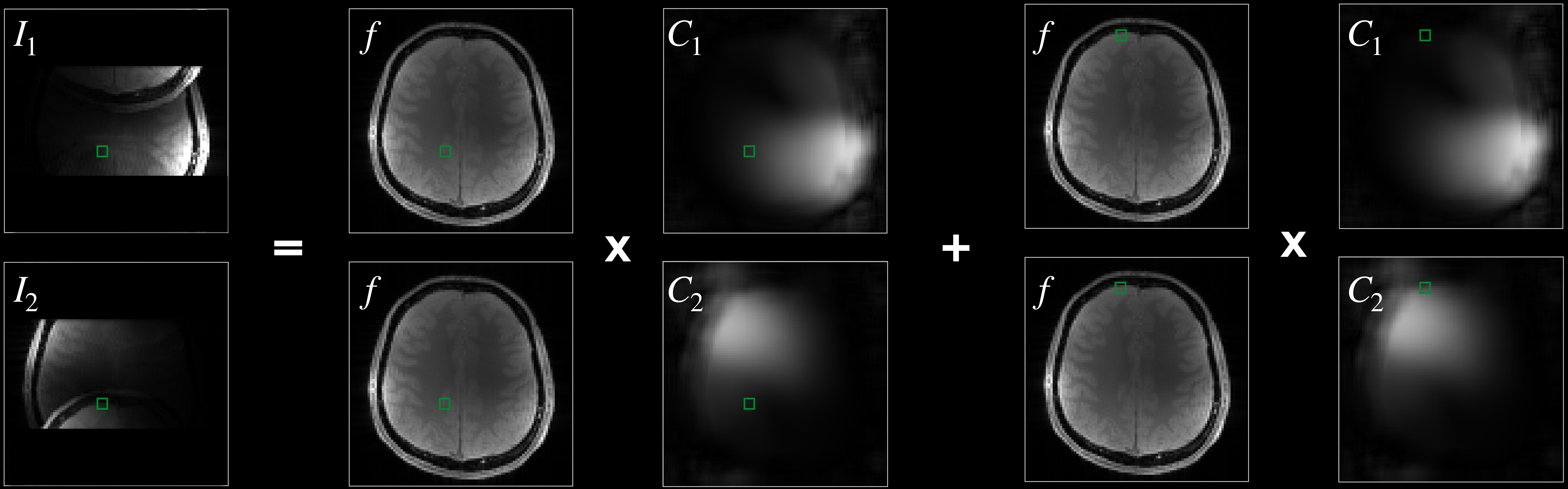
$$I_1(y) = f(y)C_1(y) + f(y + \Delta y)C_1(y + \Delta y)$$

$$I_2(y) = f(y)C_2(y) + f(y + \Delta y)C_2(y + \Delta y)$$

$$\Delta y = \frac{FOV}{R}$$

Pruessmann et al., 1999

SENSITIVITY ENCODING (SENSE)



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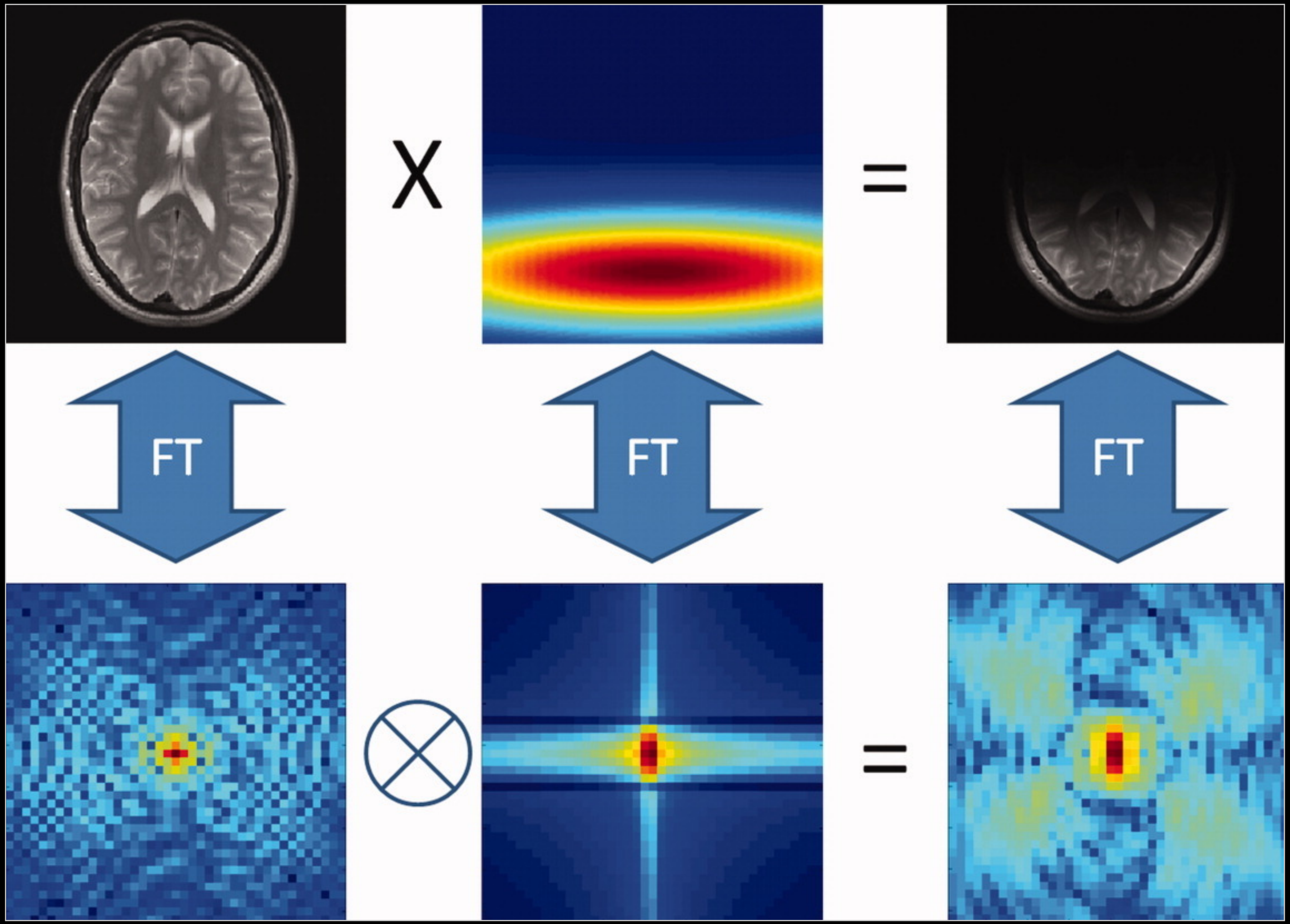
$$\Delta y = \frac{FOV}{R}$$

- prescan data
- fully-sampled calibration data
- low-rank methods (ESPIRIT,...)

Pruessmann et al., 1999

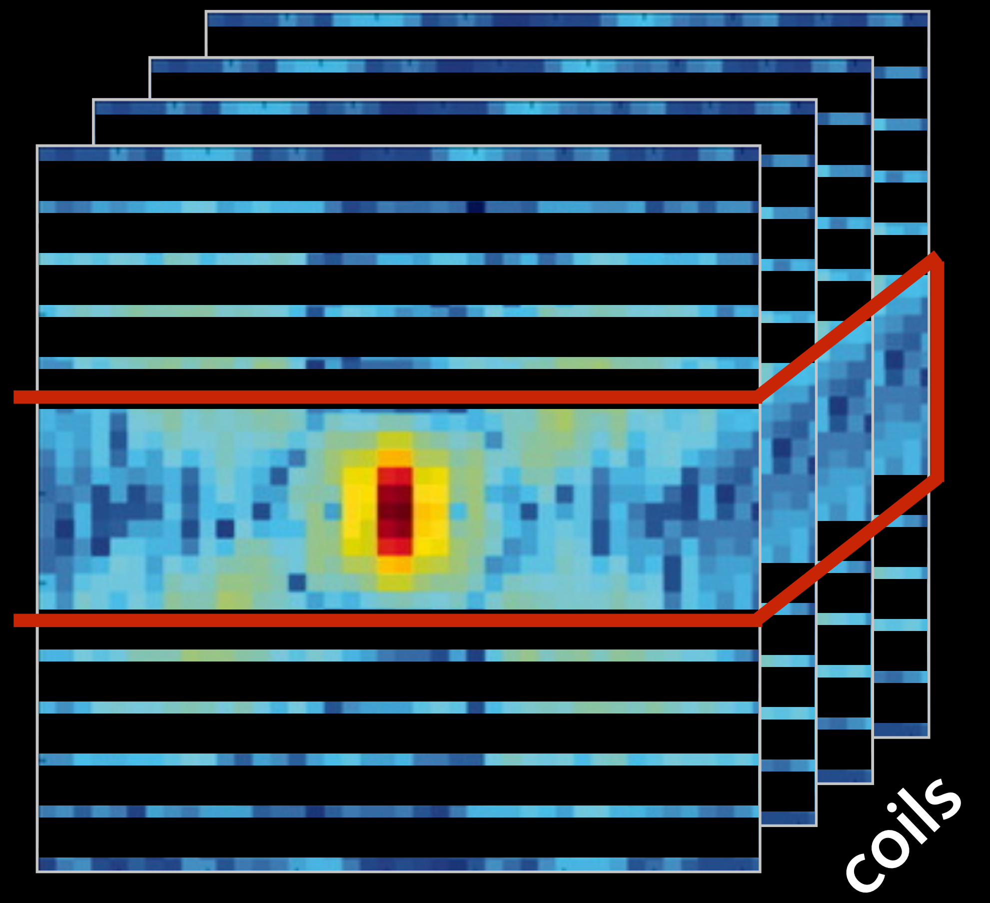
GENERALIZED AUTOCALIBRATING PARTIALLY PARALLEL ACQ.



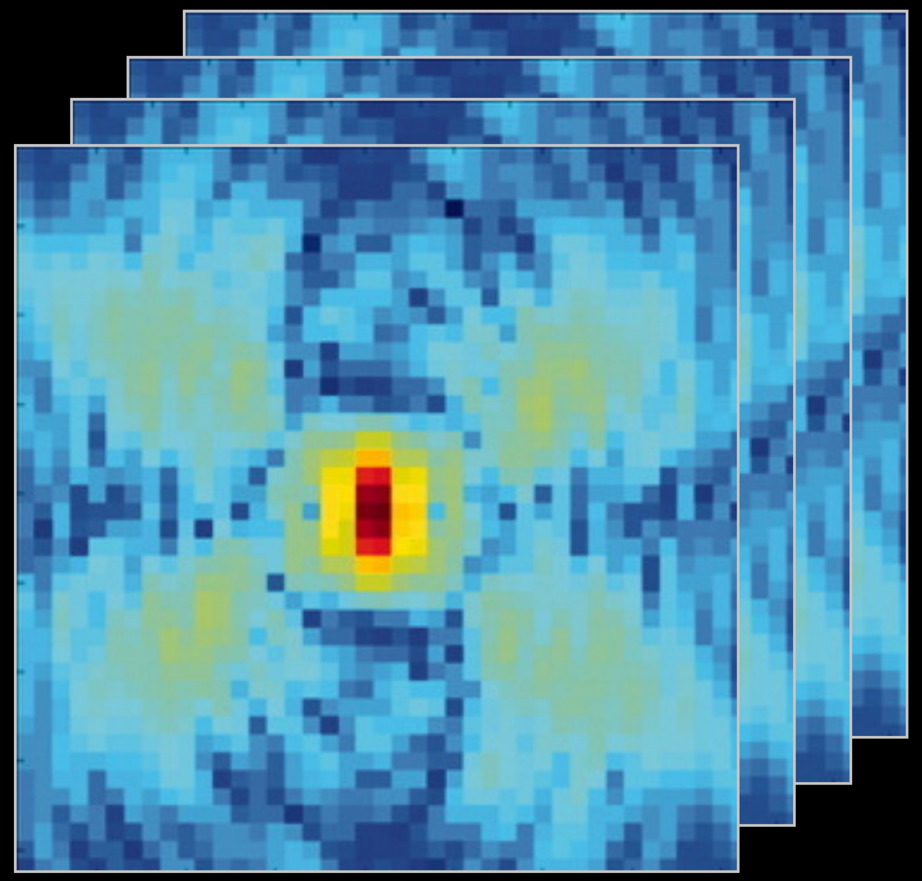
Deshmane et al., 2012

GENERALIZED AUTOCALIBRATING PARTIALLY PARALLEL ACQ.

1) estimate interpolating kernels using autocalibrating data



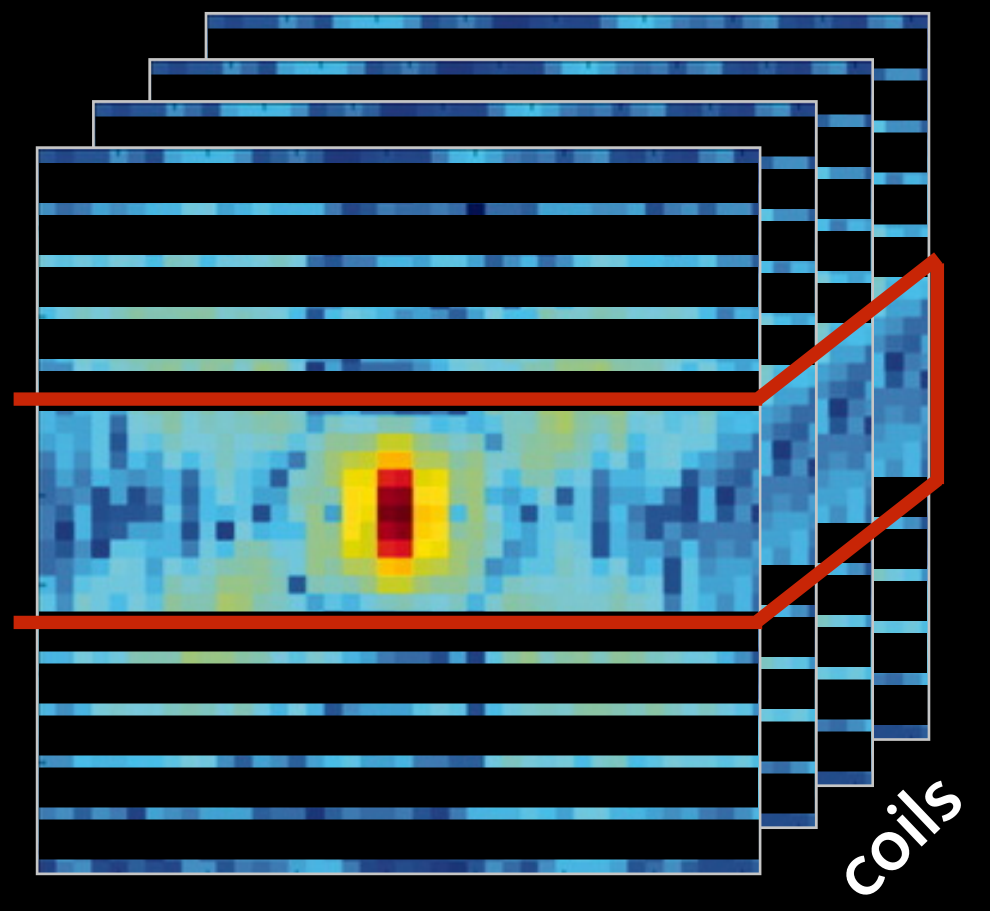
2) interpolate missing data



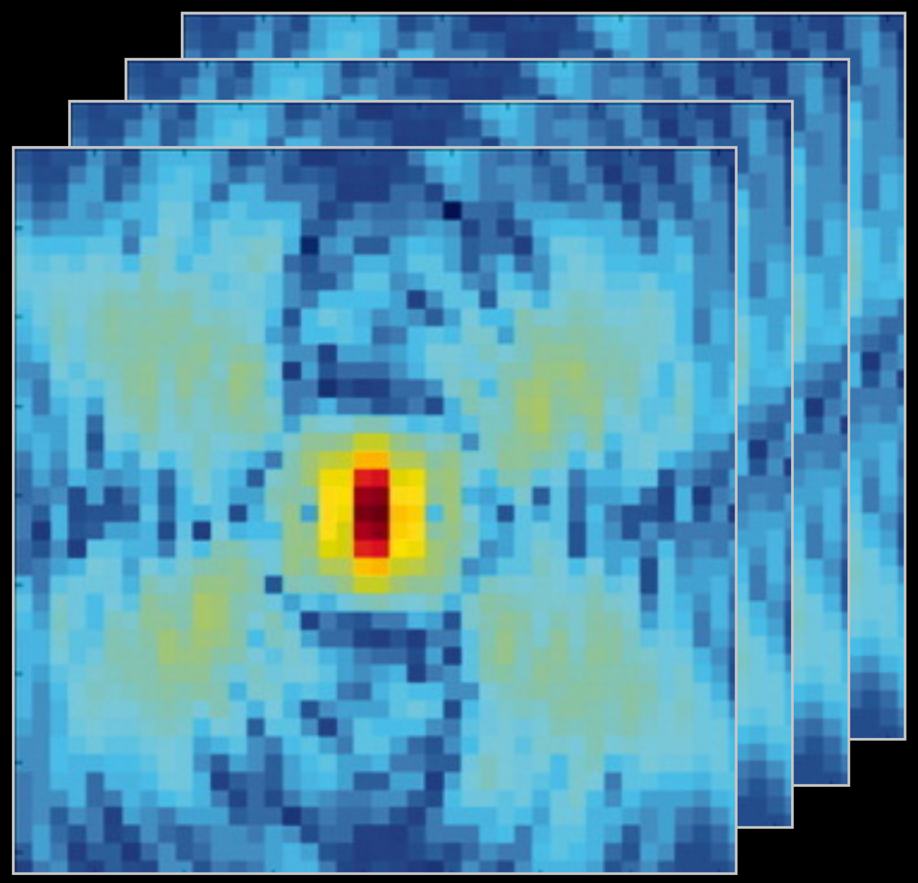
3) reconstruct and combine

GENERALIZED AUTOCALIBRATING PARTIALLY PARALLEL ACQ.

1) estimate interpolating kernels using autocalibrating data



2) interpolate missing data



3) reconstruct and combine

- kernel dimensions
- interpolation specifics
- calibration fidelity

SUMMARY

- What is k-space? How is it related to the object magnetisation?
- Reconstruction is the solution to the encoding model
- How to deal with non-cartesian sampling
- Accelerated MRI
  - Partial Fourier
  - Parallel imaging

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